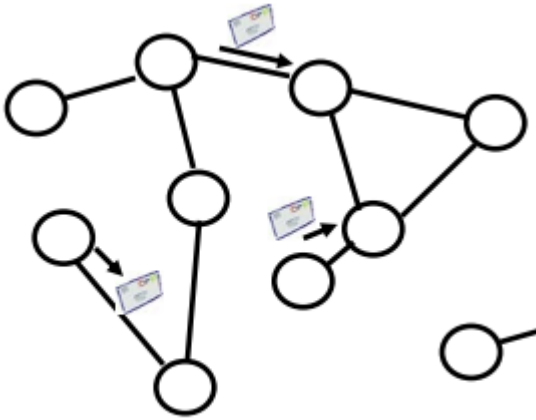
Distributed Synchronization

«A man with one clock knows what time it is –
a man with two is never sure.»

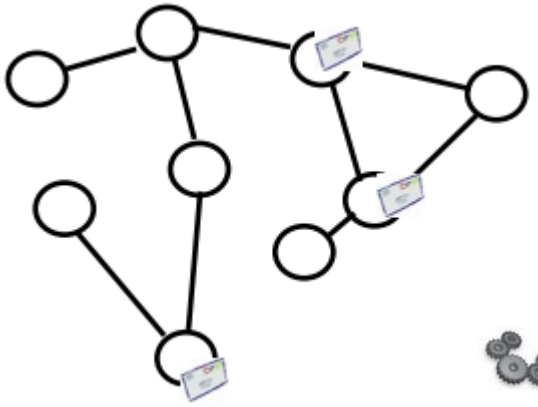
Synchronous vs Asynchronous

Synchronous algorithms simple to design and reason about:

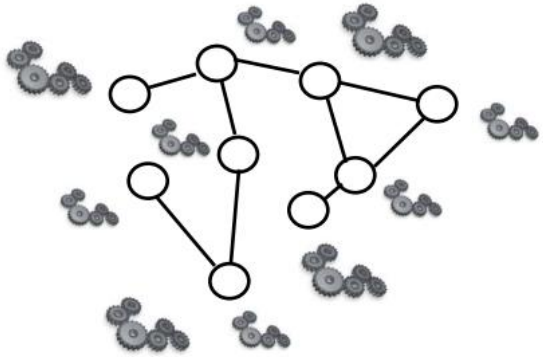
Send...



... receive...



... compute.

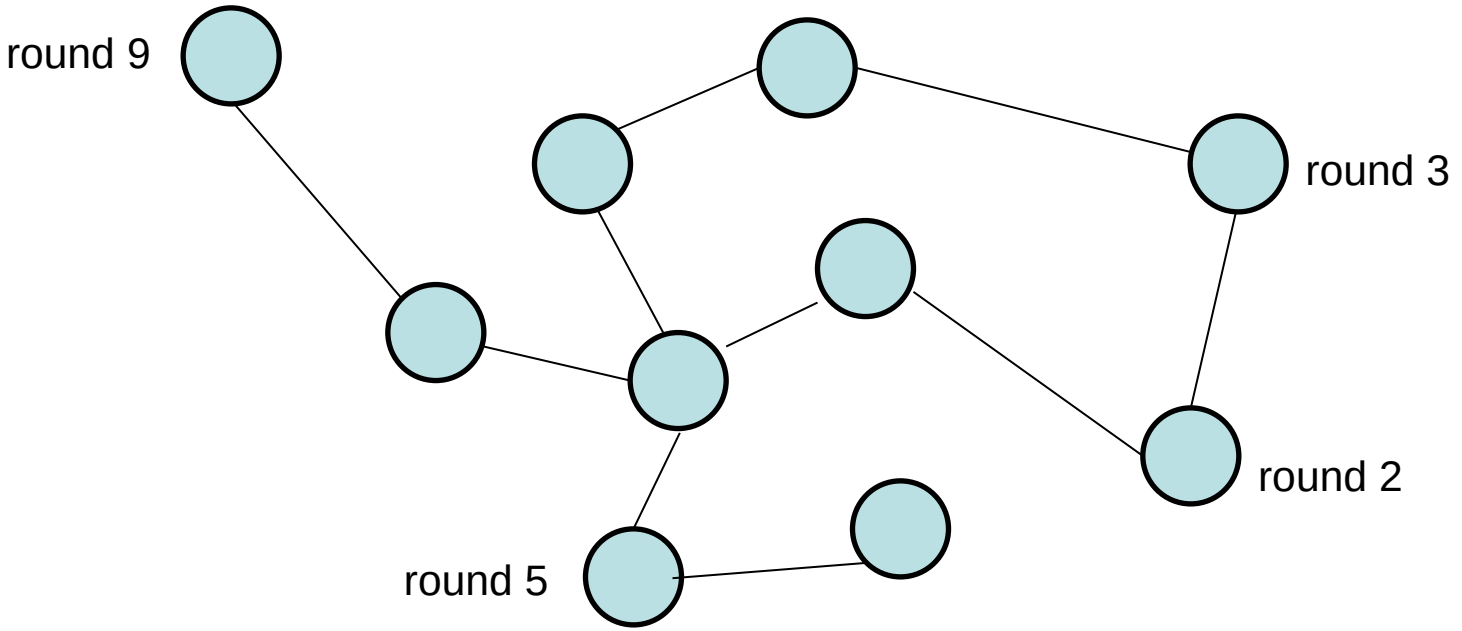


Synchronous vs Asynchronous

Asynchronous systems hard:

- Remember BFS? Agree when next phase, counters, …

Idea: How to emulate synchronous in asynchronous?



?!

Simple Idea

Local Synchronization

In round i:

- send message to all neighbors including usual packets of round i (of sync. algo, if any) plus also include round information
- go to next round when all neighbors have sent round i infos too

Idea of α-synchronizer: works with clock pulses!

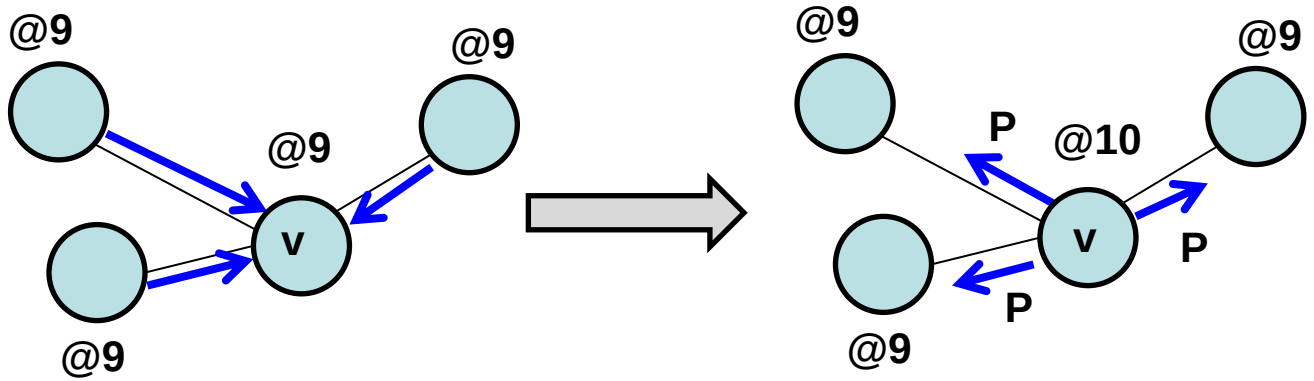
Definitions

Synchronizer

A (distributed) synchronizer generates clock pulses (PULSE) at each node of the network.

Valid Clock Pulse

A pulse generated at some node v is valid, iff it is generated after v received all the messages of the synchronous algorithm sent to v by its neighbors in the previous pulses.

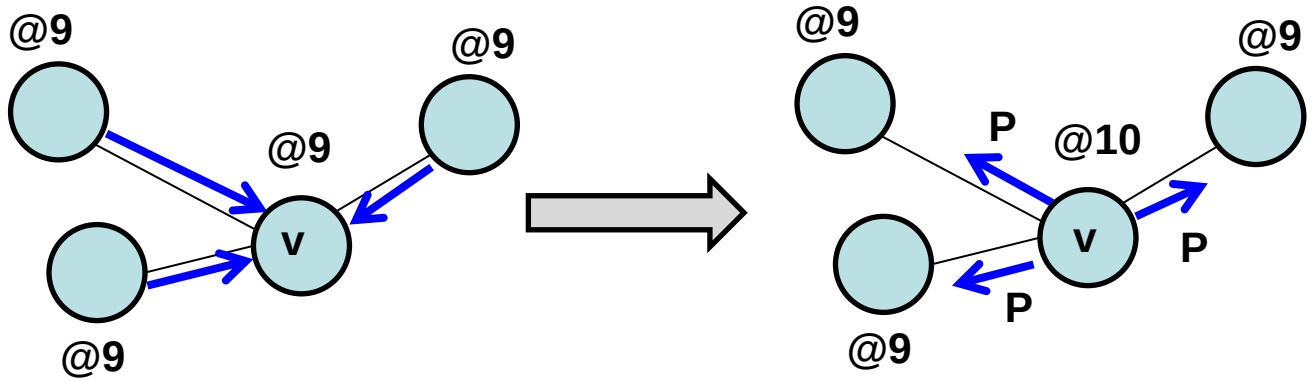


Synchronizer

A (distributed) synchronizer generates clock pulses at each node of the network.

What is overhead?
When to generate valid pulses?

synchronous algorithm sent to v by its neighbors in the previous pulses.



Overhead of Using Synchronizers

Overhead: Message needed for each pulse, independent of protocol data!

Overhead

Say T(A) and M(A) are time and message complexity of synchronous algorithm A, and T(S) and M(S) are complexities of a synchronizer for each pulse. Moreover, $T_{init}(S)$ and $M_{init}(S)$ to set up synchronizer.
Then:

$$T = T_{init}(S) + T(A)*(1+T(S)),\ M = M_{init}(S) + M(A) + T(A)*M(S)$$

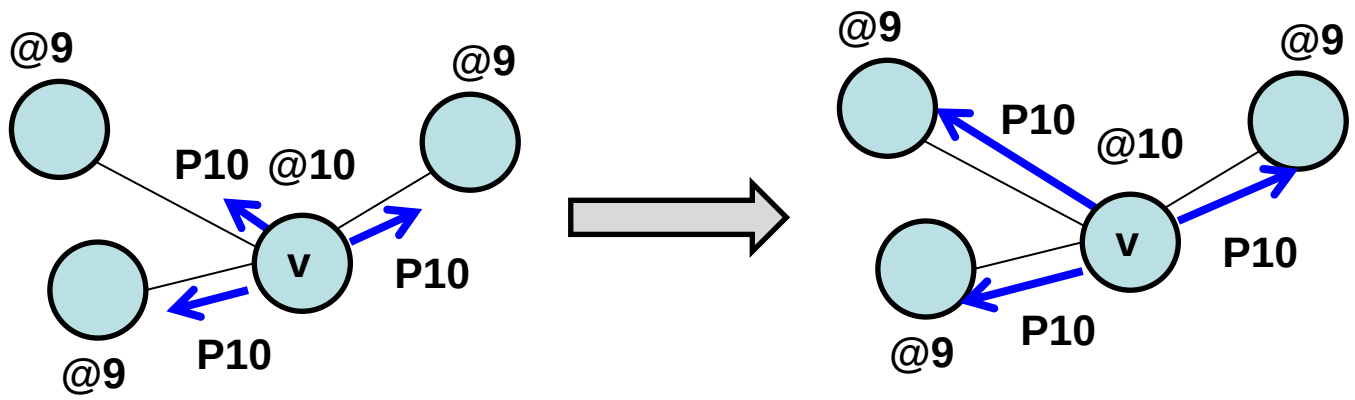
Proof.

- Setup synchronizer clear: one time cost
- Each round additionally costs time T(S) and messages M(S)

In the following, we will ignore initialization costs. ■

Safe Node

A node v is safe wrt certain clock pulse if all messages of the synchronous algorithm sent by v in that pulse have already arrived *at their destination*.



v not safe wrt 10

v safe wrt 10

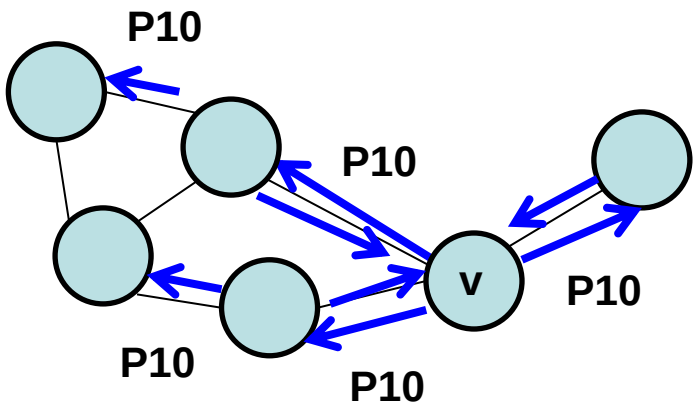
Next Pulse

If all neighbors of a node v are safe wrt the current clock pulse of v, the next pulse can be generated for v (valid!).

Proof.

If all neighbors of v are safe wrt that pulse, v has received all messages of the given pulse! So if v generates a new pulse now, it must be valid!

■



How to detect?

ACKs (reached dest!), at most factor 2 more messages

The Local Synchronizer α

Synchronizer α

At node v:

wait until v is safe (learn via ACKs)
send SAFE to all neighbors
wait until v receives SAFE messages from all neighbors
start new PULSE

Via ACKs, node detects when it is safe (= all its messages arrived at destination), the reports it to neighbors.
When all neighbors are safe (= all their neighbors including v got their messages), it continues.

No initialization needed!

Overhead?

Analysis of Local Synchronizer α

Synchronizer α

Overhead per synchronous round:

$T(\alpha) = O(1)$

$M(\alpha) = O(m)$

Why?

- Communication only between neighbors
- As soon as all neighbors of a node v become safe, v will know after one additional round
- Every edge sees at most 6 messages (PULSE, SAFE, ACK) (but unfortunately, maybe in reality no message would have been sent in sync algo)

Happy?

Analysis of Local Synchronizer α

Synchronizer α

Overhead per synchronous round:

$T(\alpha) = O(1)$

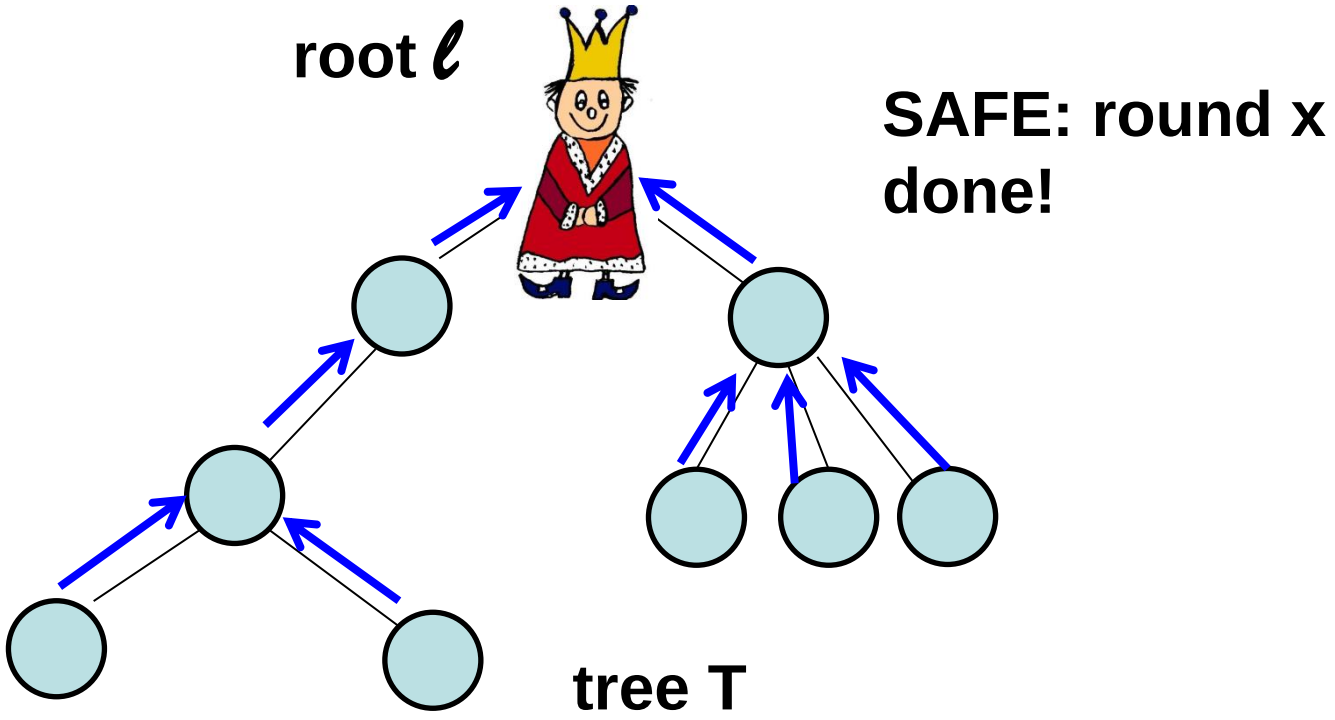
$M(\alpha) = O(m)$

Time complexity good, but too much communication!

Ideas? Recall «Flooding»...

The Global Synchronizer β

Idea: make global synchronization:
Pre-compute a **leader** and a **spanning tree**: convergecast to next phase!



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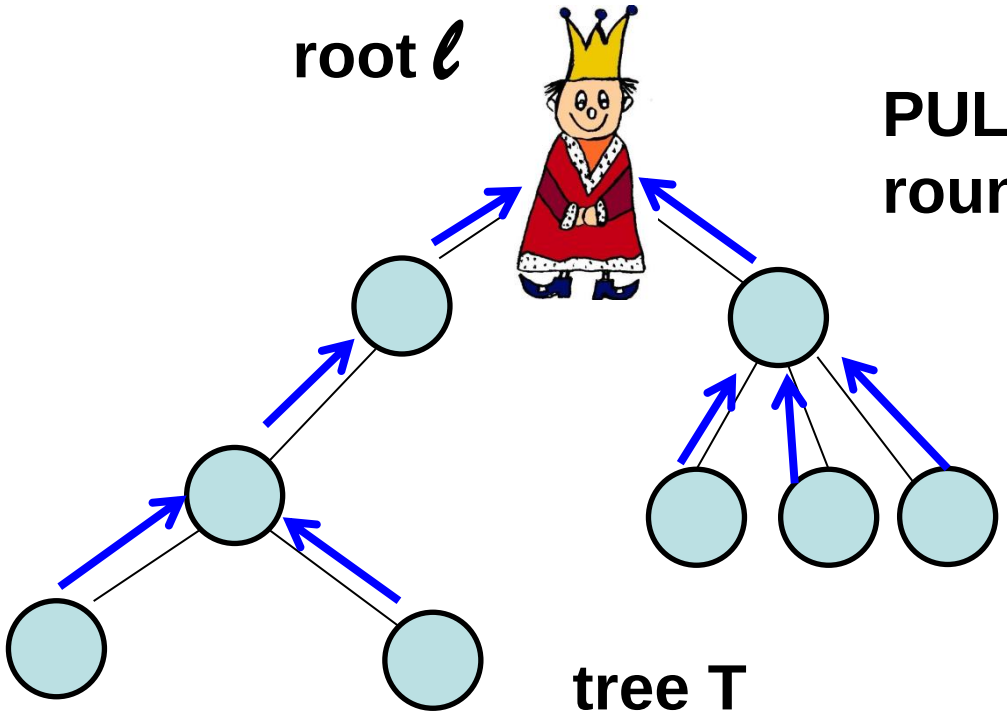


The Global Synchronizer β

Idea: make global synchronization:
Pre-compute a **leader** and a **spanning tree**: convergecast to next phase!

Synchronizer β

At node v:

wait until v is safe
wait until v receives SAFE message from all its children in tree
send SAFE message to parent in T
wait until PULSE received from parent
send PULSE to children
start PULSE

Complexity?

Synchronizer β

Complexities per synchronous round:

$T(\beta) = O(\text{diam } T) = O(n)$

$M(\beta) = O(n)$

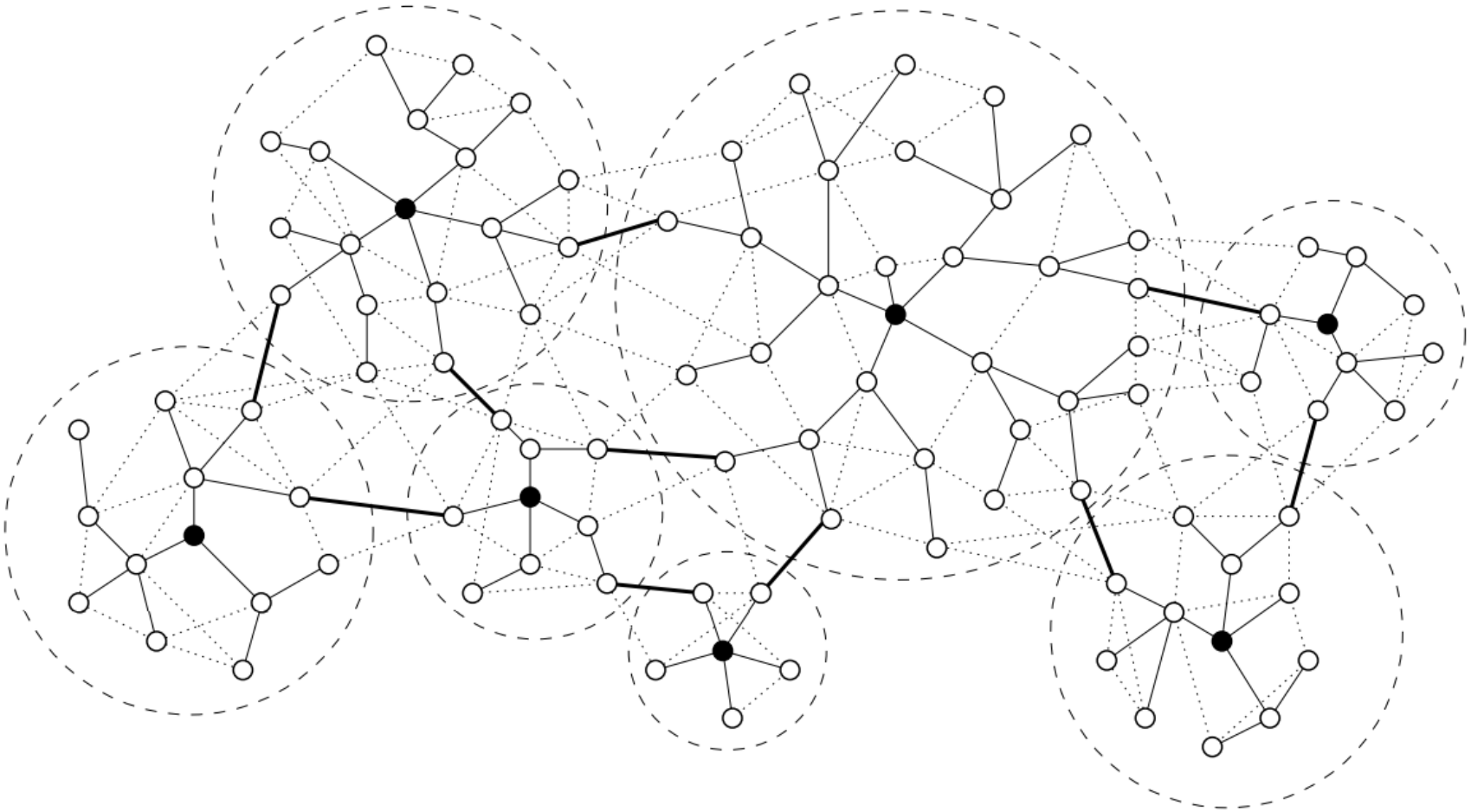
Why?

- $T(\beta)$ and $M(\beta)$: follow from convergecast
- Initialization: improved GHS gives $T_{init}(\beta) = O(n)$, $M_{init}(\beta) = O(m + n \log n)$

Local synchronizer faster, global synchronizer less messages!
How to combine?

The Hybrid Synchronizer γ

Idea: Make β in dense areas and α in sparse areas



The Hybrid Synchronizer γ

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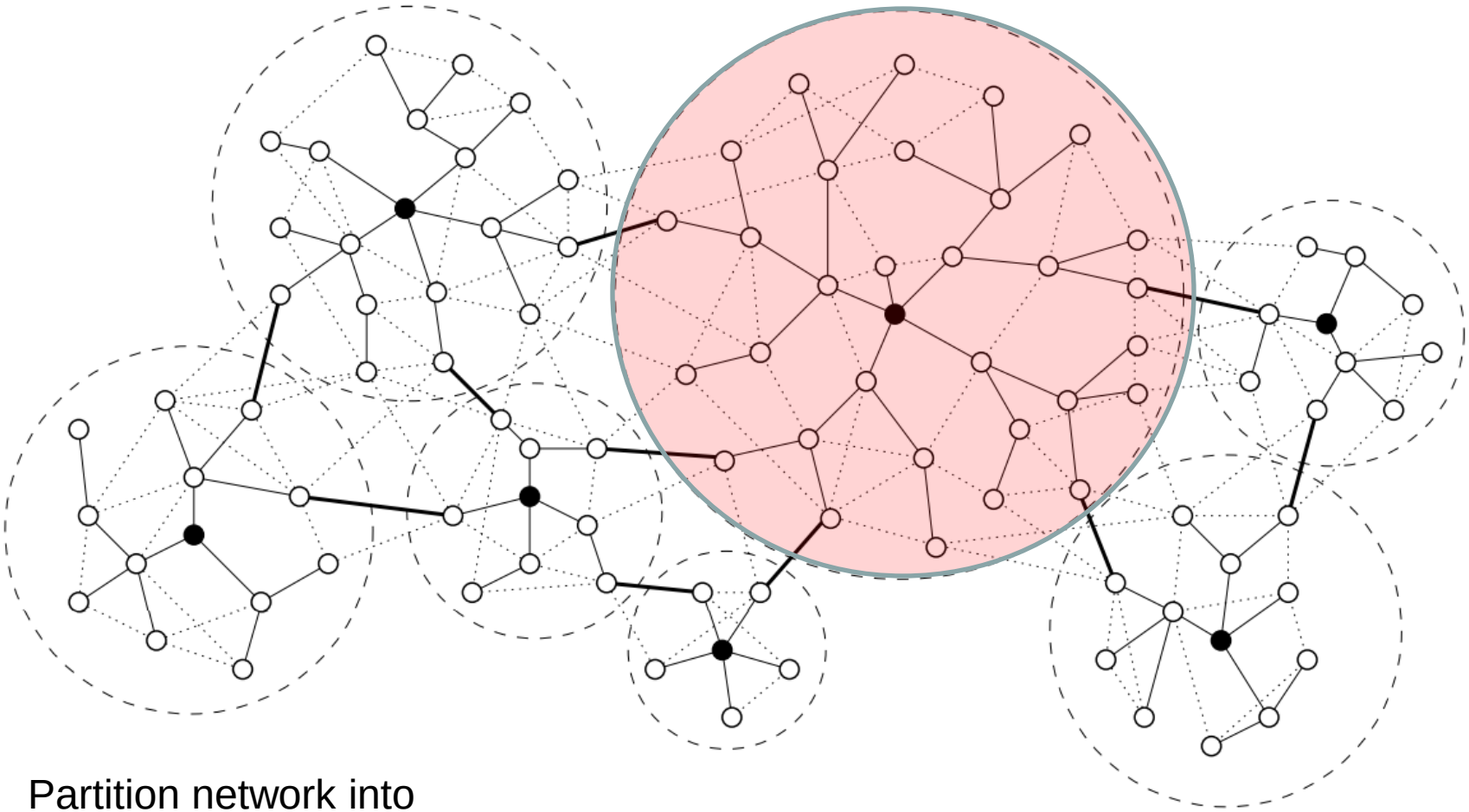
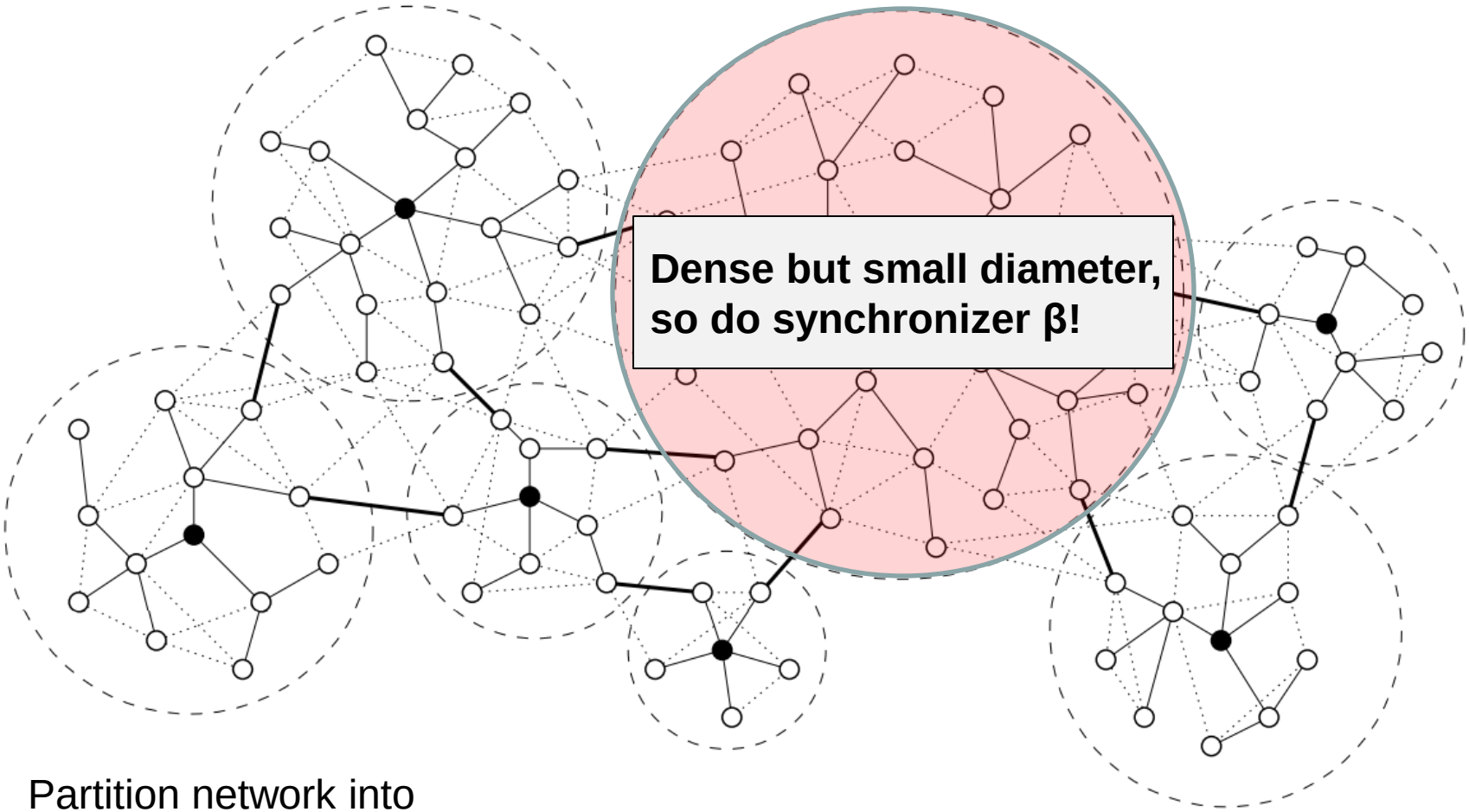


Partition network into
small-diameter clusters.

The Hybrid Synchronizer γ

Idea:



Partition network into
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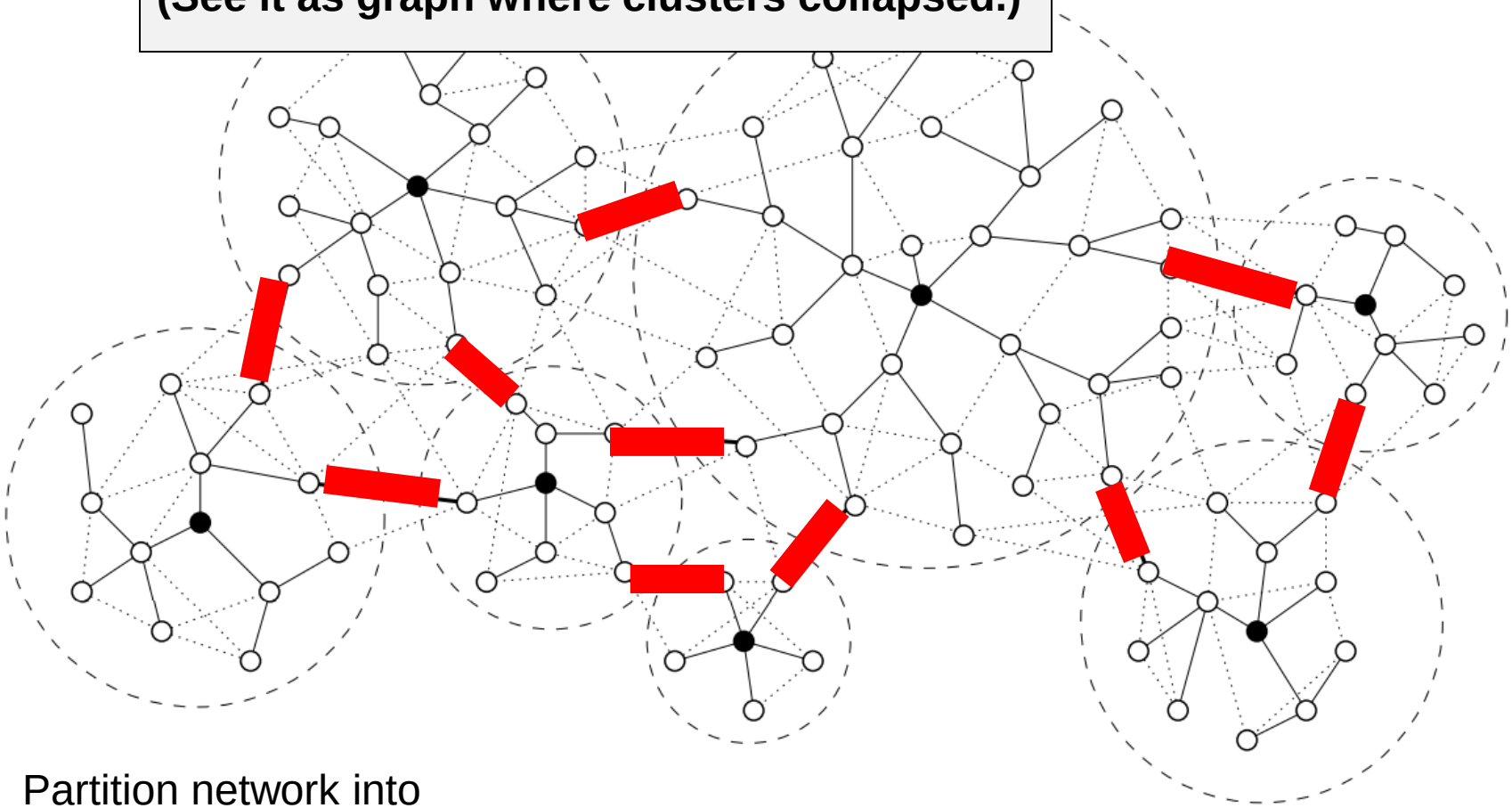
The Hybrid Synchronizer γ

Idea:

Between clusters, do synchronizer α!
(See it as graph where clusters collapsed.)



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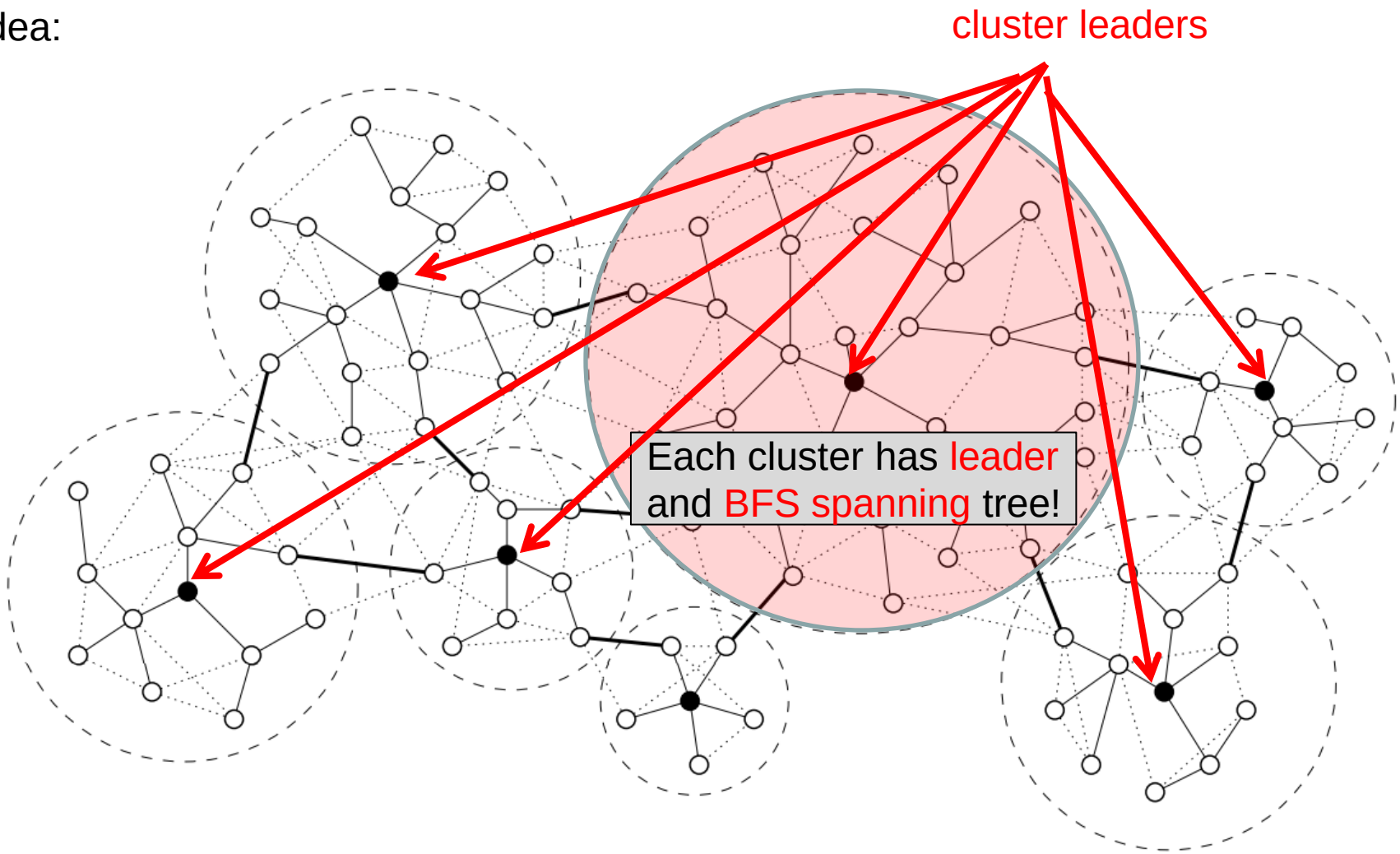

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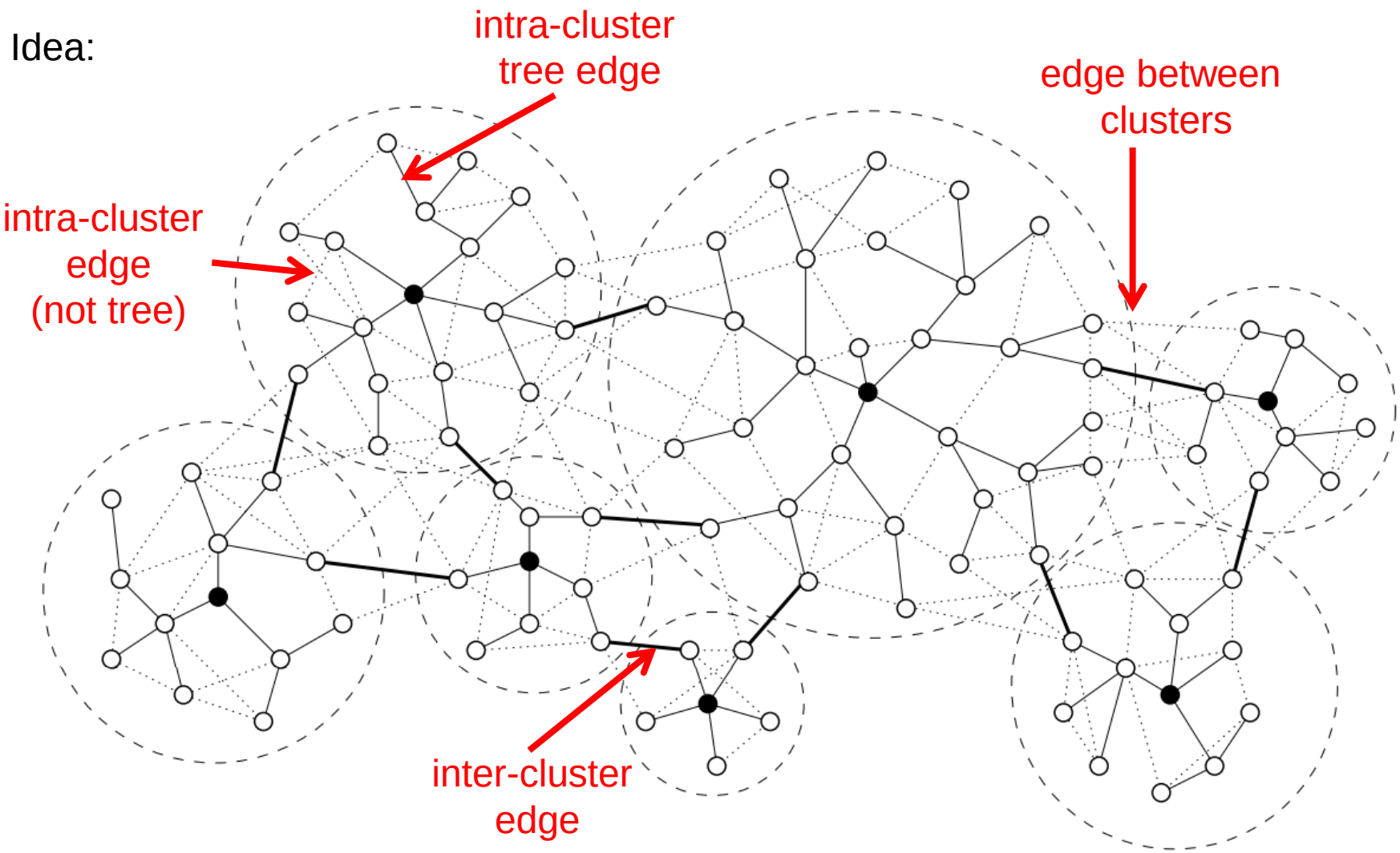

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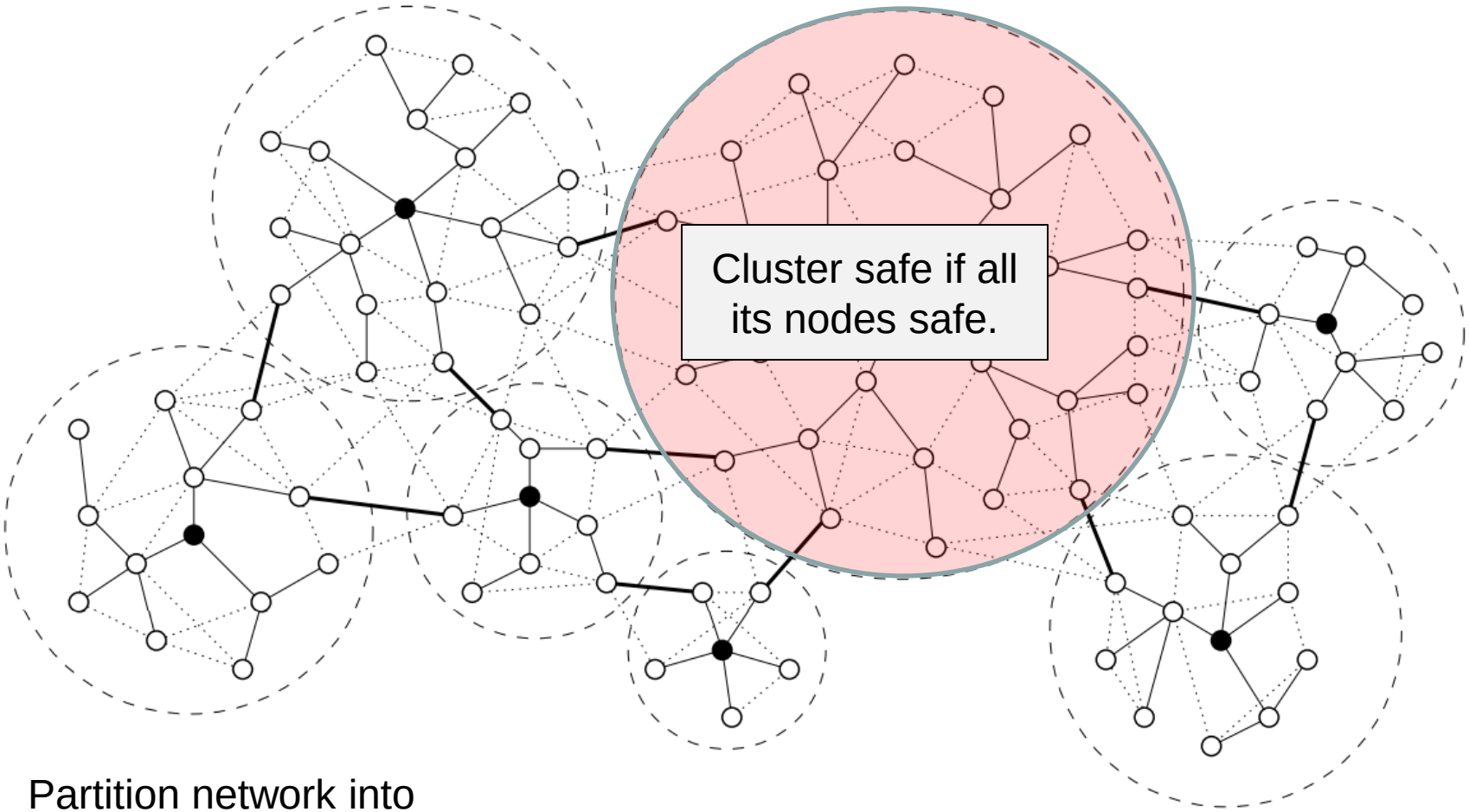

Partition network into small-diameter clusters.

The Hybrid Synchronizer γ

Idea:

1. Phase 1: Apply Synchronizer **β** in each cluster;
   when done inform leaders in neighbor clusters
2. Phase 2: Generate next pulse when neighbor clusters are safe (Synchronizer α)

Synchronizer γ

For node v:

wait until v is safe
wait until v receives SAFE from all children in intra-cluster tree
send SAFE to parent in tree
wait for CLUSTERSAFE message from parent
send CLUSTERSAFE to children
wait until NEIGHBORSAFE received from all incident
inter-cluster edges and children in intra-cluster
send NEIGHBORSAFE to parent
wait for PULSE and forward

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Complexity?

Synchronizer γ

Let m_c be number of inter-cluster edges and let k be the maximum cluster radius (max dist leaf to leader).
Then:

$$T(\gamma) = O(k)$$
$$M(\gamma) = O(n+m_c)$$

Why?

Synchronizer γ

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Why?

- Time:
  - Depth of tree enough (among all neighbor clusters).
- Messages:
  - intra-cluster tree edges see SAFE, CLUSTERSAFE, NEIGHBORSAFE, PLUSE, plus ACKs. It's trees, so O(n).
  - one NEIGHBORSAFE for each inter-cluster edge

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Want to minimize k and m_c: How to partition?

Possible Partitions

- What happens if each single node is its own partition?
- What if everything is one big cluster?

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What's our synchronizers!

Network Partition

Idea:

1. Construct one cluster after another; start cluster at random non-covered node
2. Grow as long as “growth significant” (factor ρ)

Define: B(v,r) = Ball of radius r around v

Cluster Construction

```
while unprocessed nodes:
        select arbitrary unprocessed node v
        r:=0
        while |B(v,r+1)| > ρ * |B(v,r)| do
                r := r+1
        end while
        makeCluster(B(v,r))
end while
```

Partition Properties

The resulting network partition:

(1) consists of clusters of radius at most $\log_\rho n$

(2) at most $(\rho - 1)*n$ intercluster edges

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Why?

- Cluster radius:
  - Radius grows only if cluster size increases by factor ρ
  - As there are at most n nodes, this can happen at most $\log_\rho n$ times
- Intercluster edges:
  - Intercluster edge connects node at border of cluster with node outside cluster
  - Consider cluster C=B(v,r): we know $|B(v,r+1)| \leq \rho * |B(v,r)|$.
  - The number of nodes adjacent to a cluster can hence be at most
    $| B(v,r+1) \setminus B(v,r) | \leq \rho * |C| - |C|$
  - There is only one intercluster edge per cluster-pair, so at most $\rho * |C| - |C|$ per cluster
  - Summing over all cluster (n nodes in total): $\sum (\rho - 1)* |C| = (\rho - 1)* \sum |C| = (\rho - 1)* n$

Quality of Partition

Partition Properties

The resulting network partition:

(1) consists of clusters of radius at most $\log_\rho n$

(2) at most $(\rho - 1)*n$ intercluster edges

Asymptotically optimal tradeoff!

Partition Properties

The resulting network partition:

(1) consists of clusters of radius at most $\log_\rho n$

(2) at most $(\rho - 1)*n$ intercluster edges

Example: $\rho = 2$

Logarithmic time, linear number of intercluster edges

Example: $\rho = n^{1/k}$

k time, $O(n^{1+1/k})$ intercluster edges

Remarks

- Try to make synchronous BFS algorithm asynchronous with γ synchronizer

- Bad about our solution: all nodes need to participate in synchronization even if they are “active” only during small algo period

End of Lecture