Network Algorithms

More Leader Elections: Consensus with Failures!

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,

Recap (1)

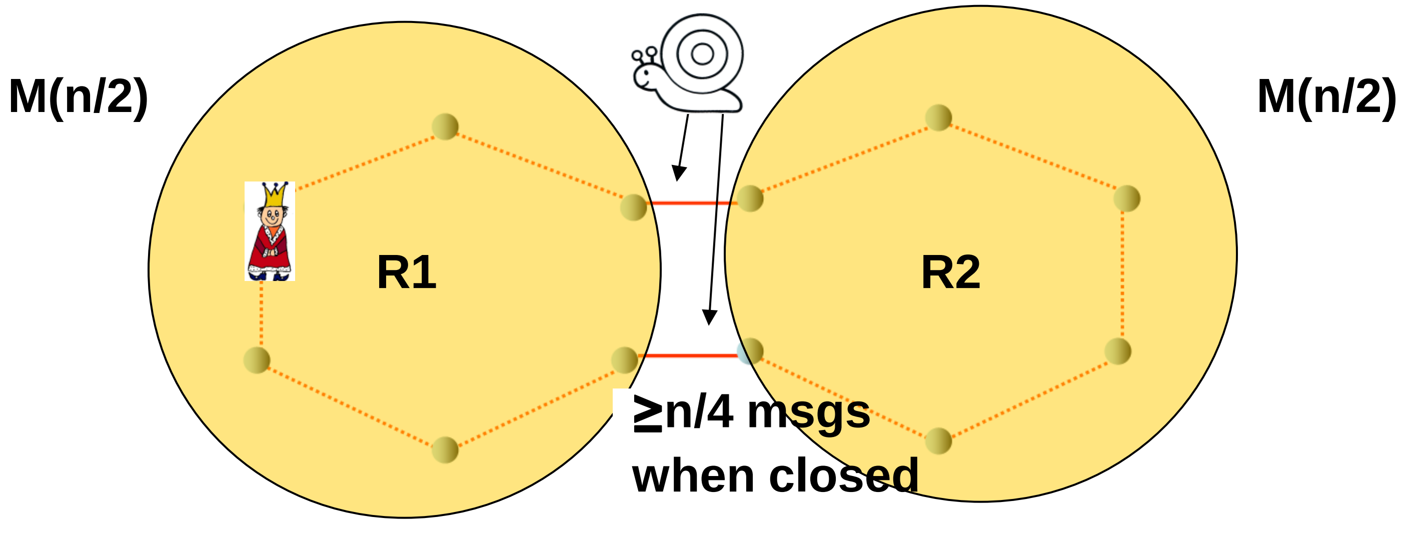
Leader election solved!

So we can solve any network problem!?

Game/Lecture over?

Recap (2)

- Leader Election and Consensus
  - When possible, when not? It depends!
- Leader Election in Networks
  - Not possible in ring without IDs
  - With IDs, in O(n) time and O(n log n) messages, even if n unknown and transmissions asynchronous
  - This is optimal: in asynchronous, need > n log n message is ring!
  - Synchronous: can do with n messages, treat messages with time!

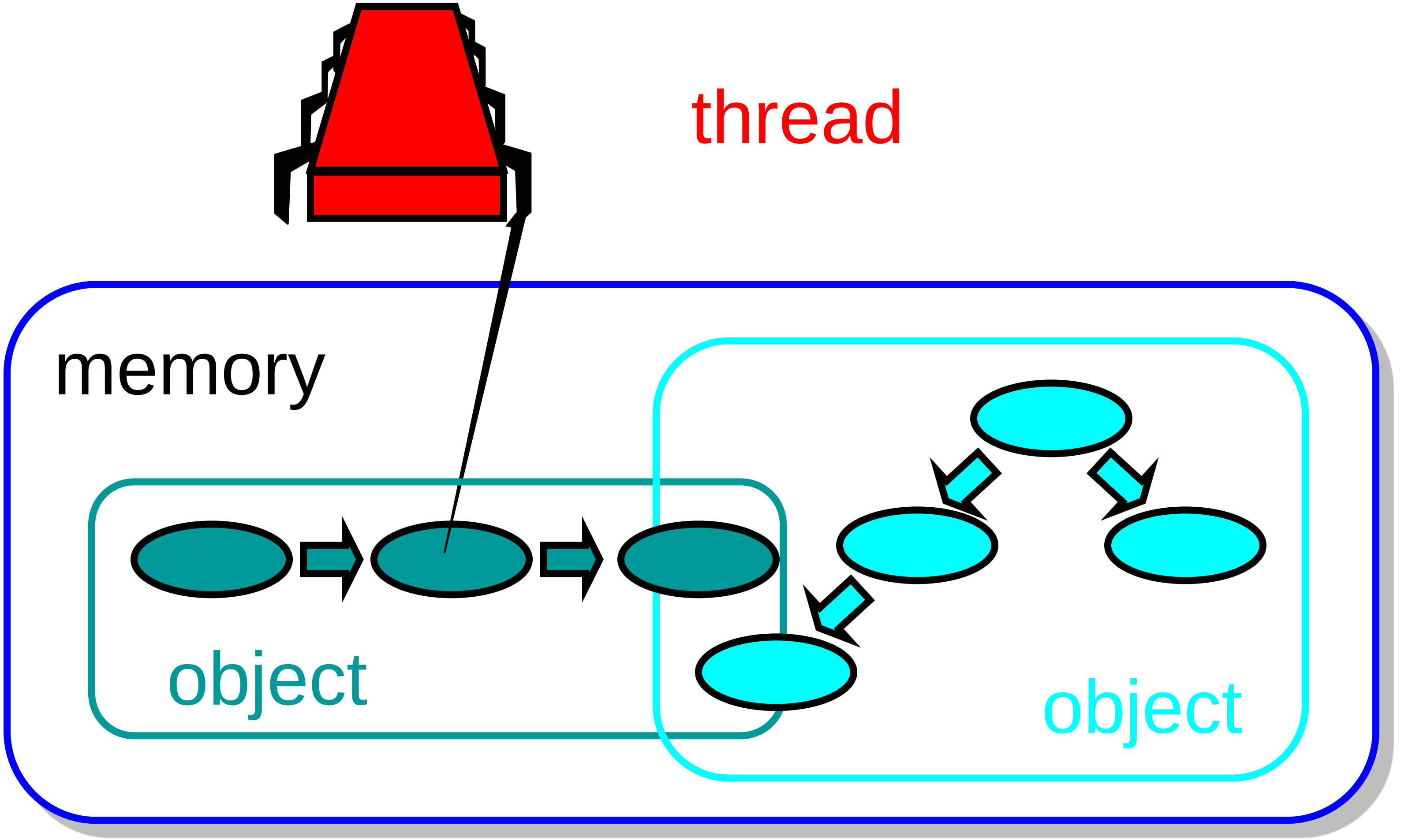


Recap (3)

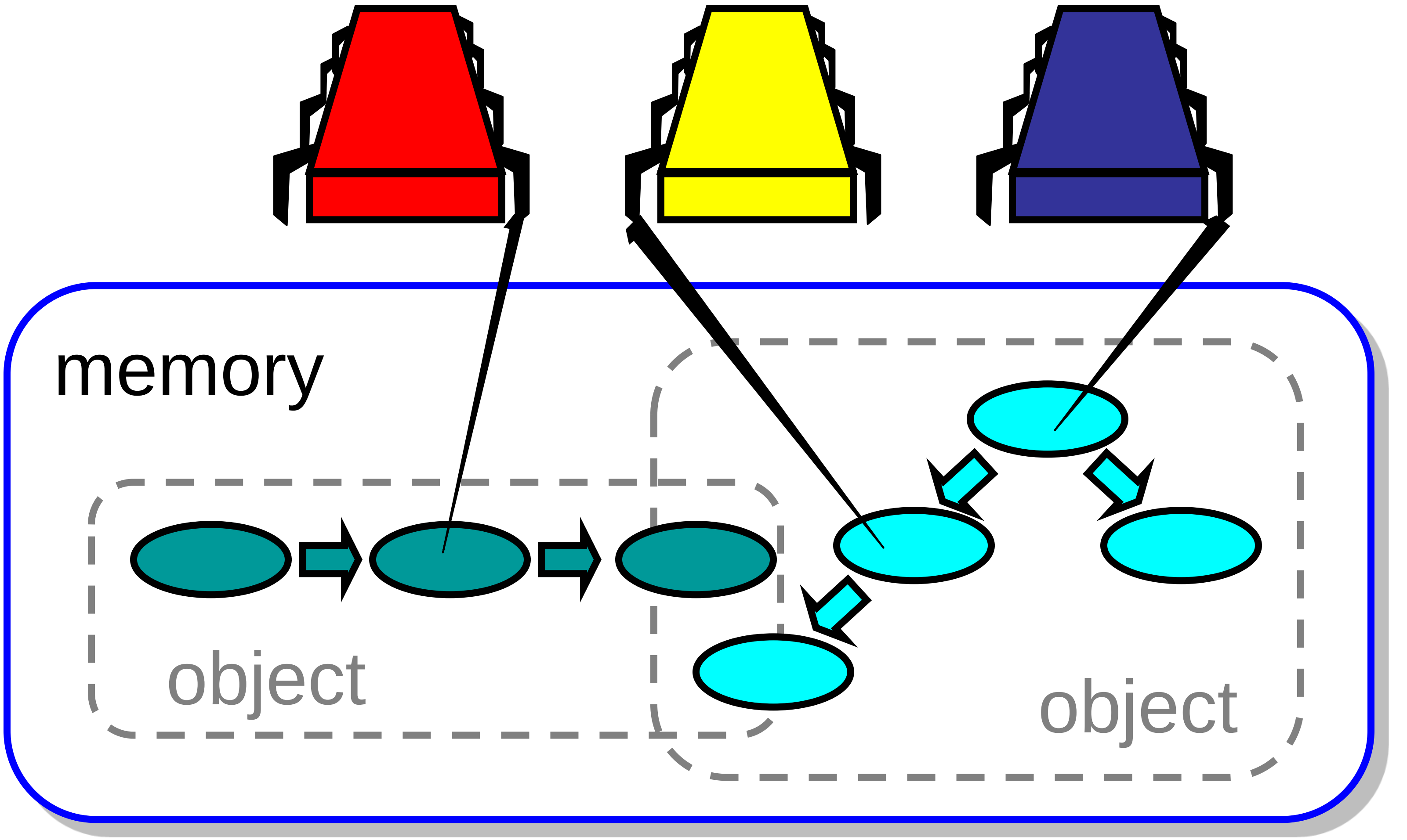
- Introducing failures: Byzantine Generals consensus impossible under message loss
  - Harder than NP-hard ☺

- Make it simpler: consensus in shared memory (no network!)
  - Consensus possible if processes do not die! Your algo: write value in my register, decide on minimum!
  - What about failures, does it work too?! If failures are fail-stop? If behavior is "malicious"? If behavior is Byzantine (arbitrary)?

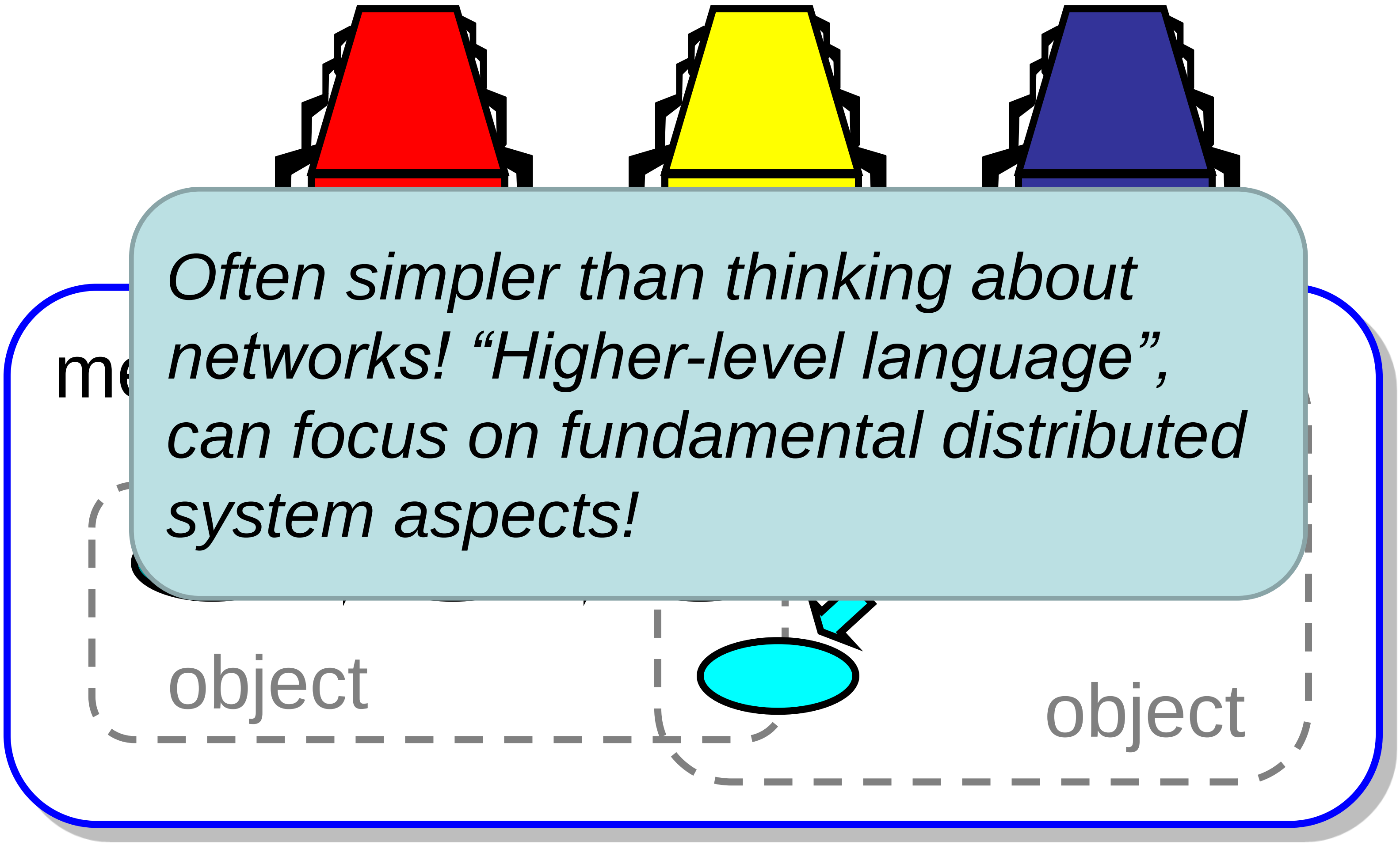
Sequential Computation



Parallel Computation

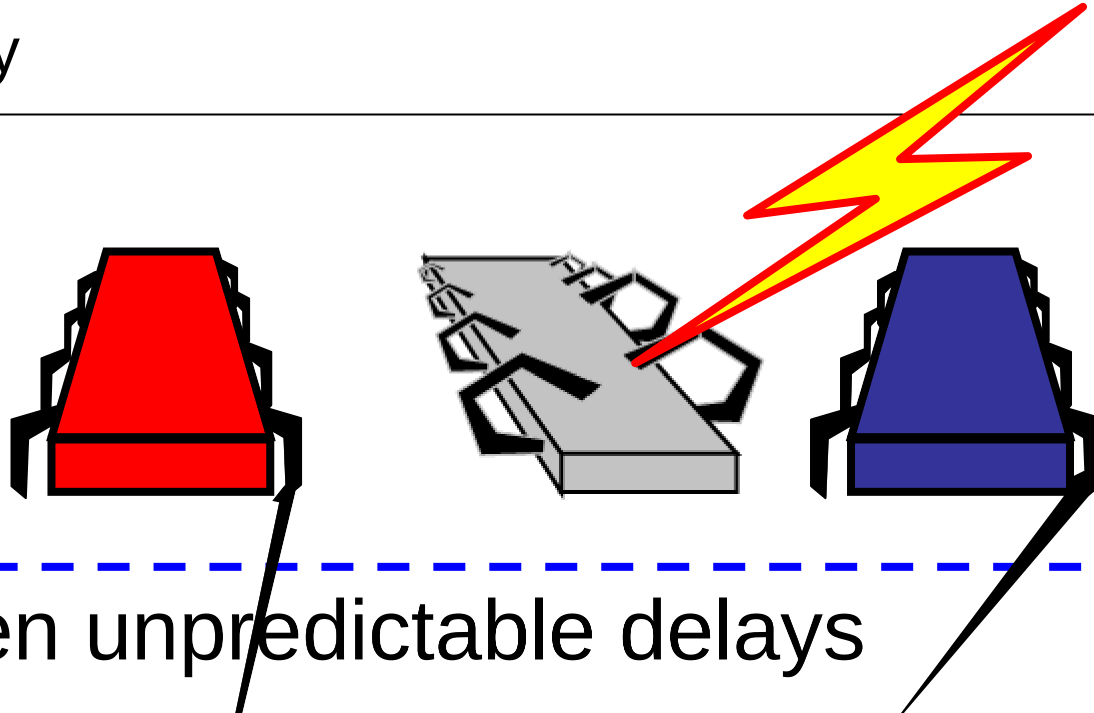


Parallel Computation



Often simpler than thinking about networks! "Higher-level language", can focus on fundamental distributed system aspects!

Asynchrony

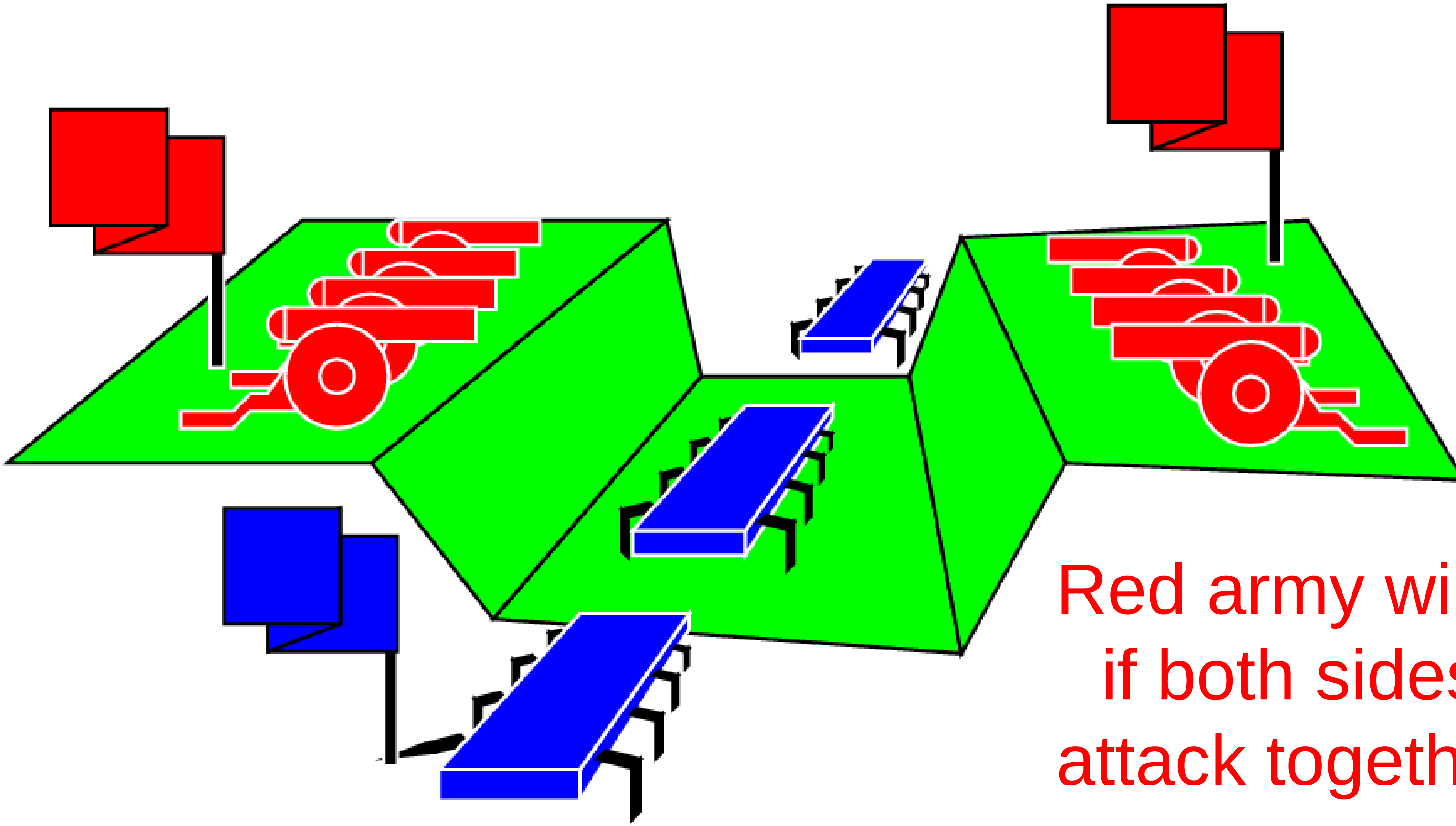


- Sudden unpredictable delays
  - Cache misses (*short*)
  - Page faults (*long*)
  - Scheduling quantum used up (*really long*)

Model

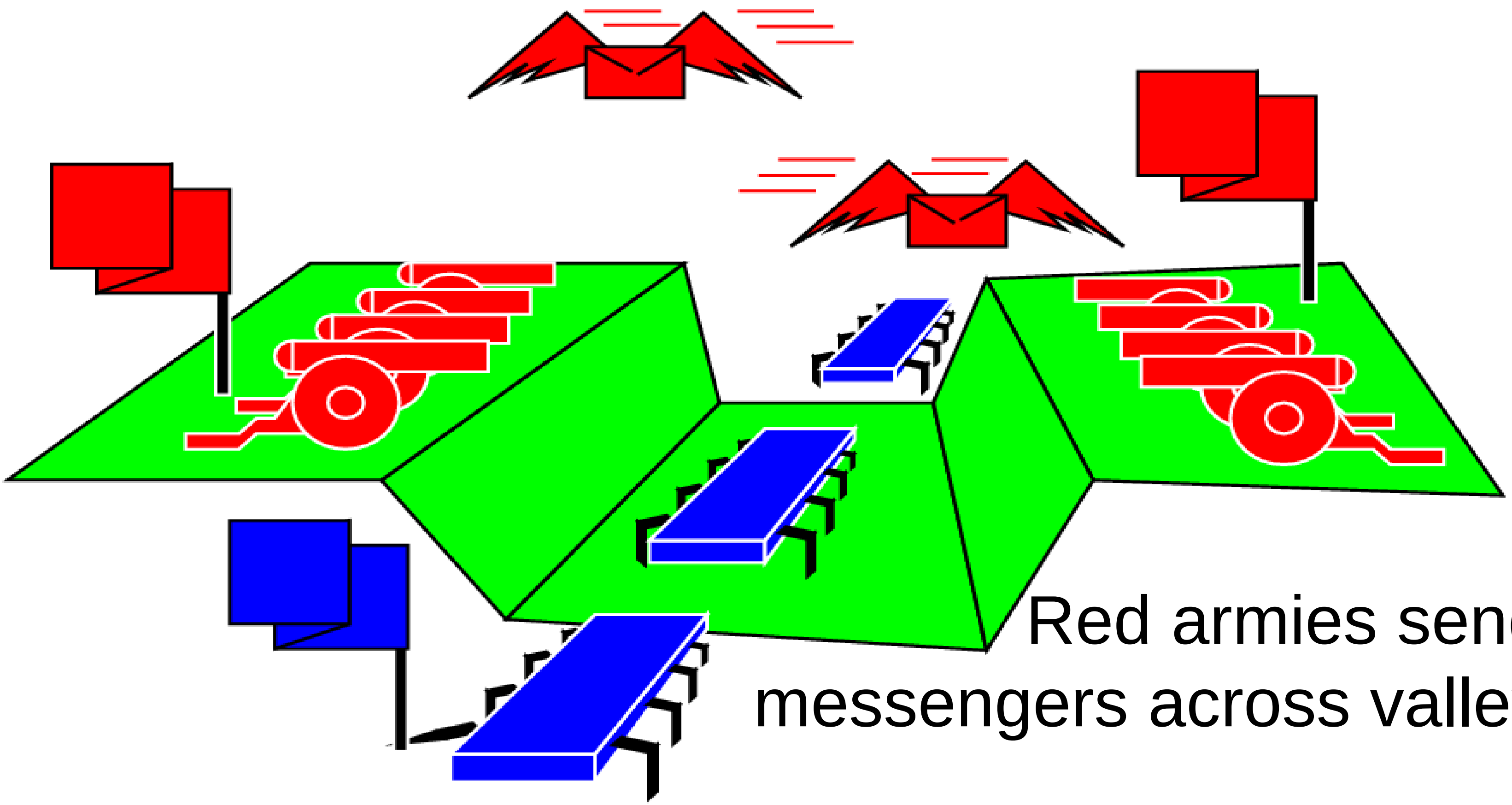
- Multiple *threads*
  - Sometimes called *processes*
- Single shared *memory*
- *Objects* live in memory
- Unpredictable asynchronous delays

Two Generals Problem



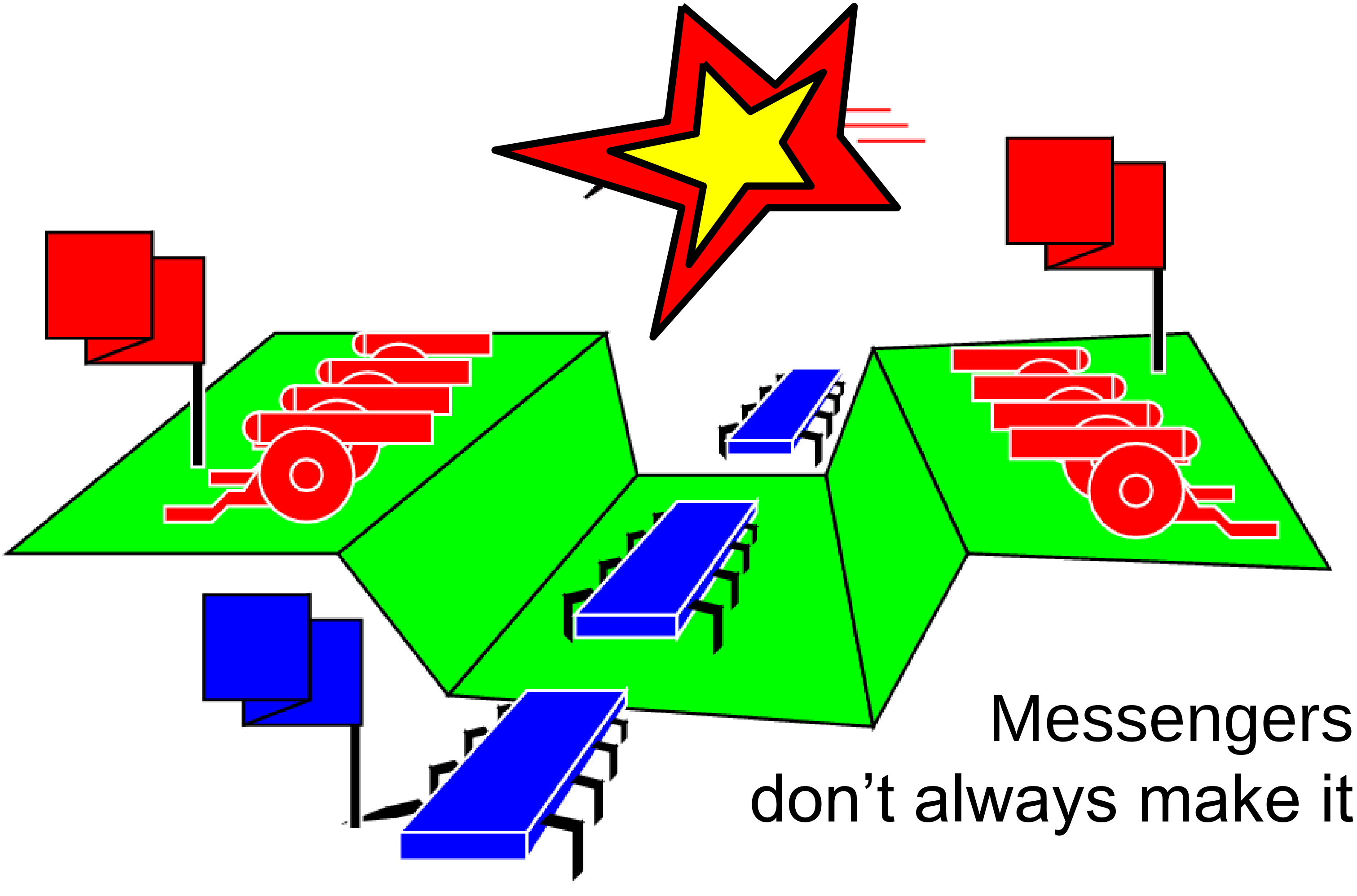
Red army wins
if both sides
attack together

Communications



Red armies send messengers across valley

Communications



Messengers
don’t always make it

Theorem

There is no non-trivial protocol that ensures the red armies attack simultaneously!

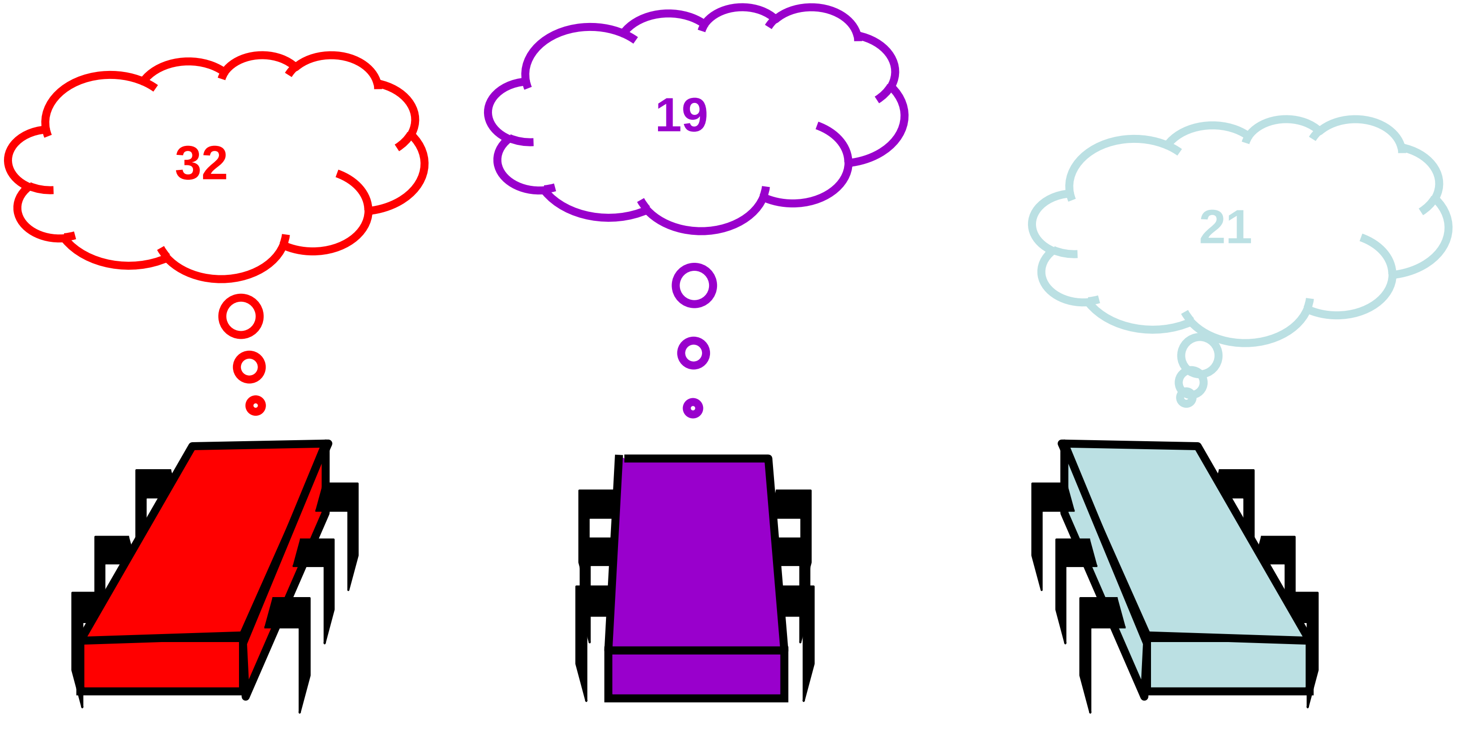
Proof Strategy

- Assume a protocol exists
- Reason about its properties
- Derive a contradiction

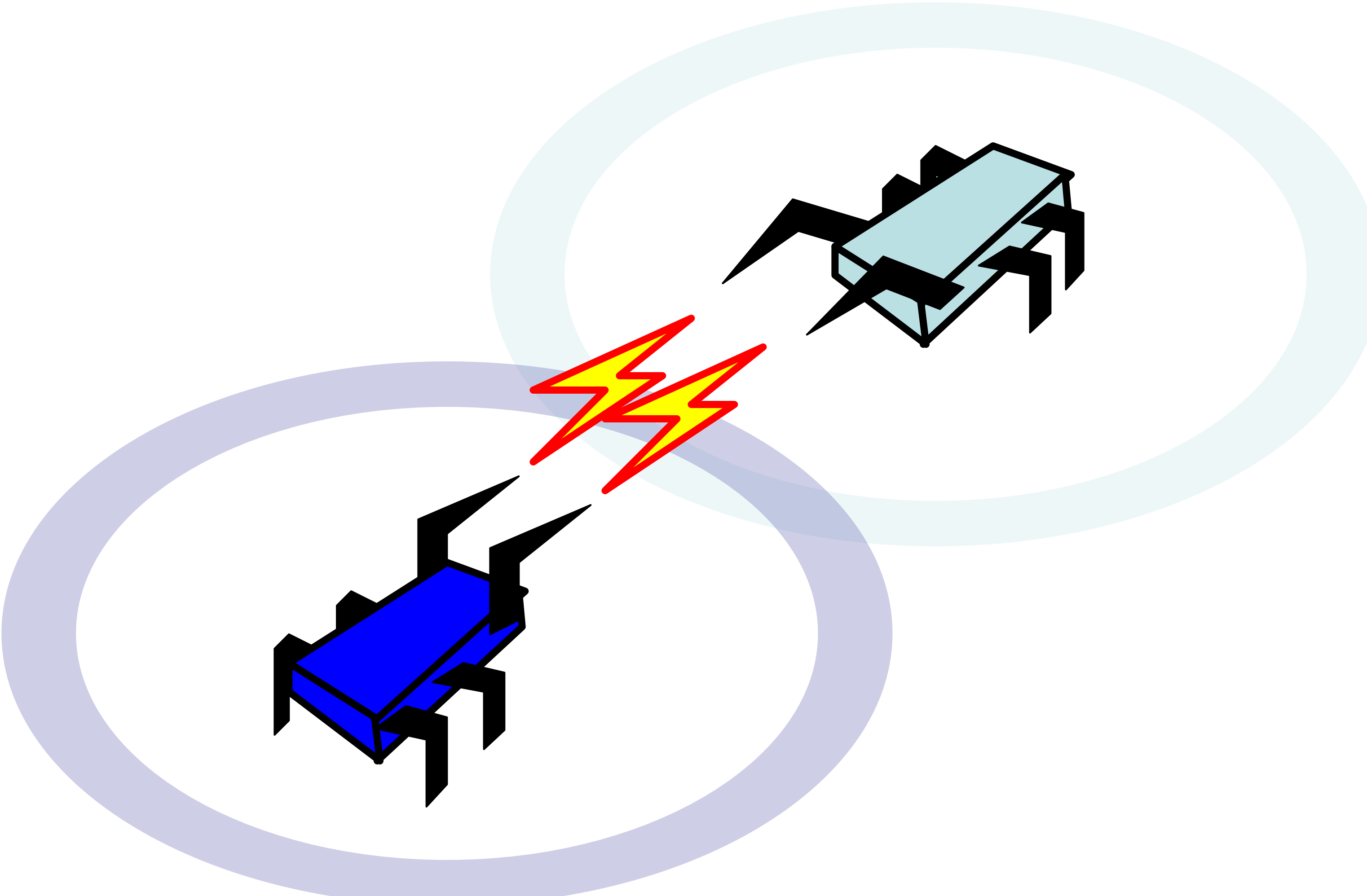
Proof

1. Consider the protocol that sends fewest messages
2. It still works if last message lost
3. So just don’t send it
   – Messengers’ union happy
4. But now we have a shorter protocol!
5. Contradicting #1

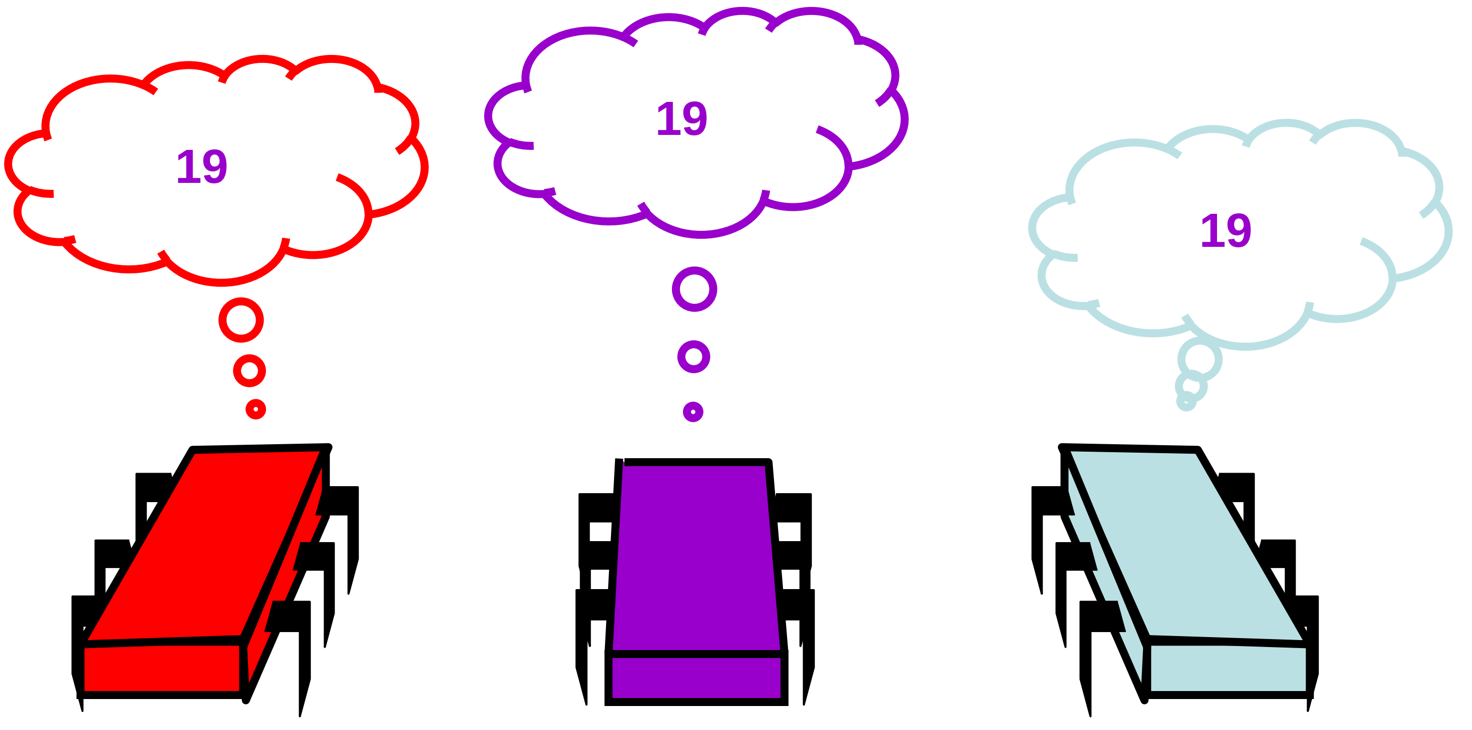
Consensus: Start...



... Communicate...



... Agree on Someone’s Input!



Why Consensus?

- With consensus, you can implement anything you can imagine…
- Examples: with consensus you can decide on a leader, implement mutual exclusion, or solve the two generals problem

What you will learn...

- In some models, consensus is possible
- In some other models, it is not

- Goal of this and next lecture: to learn whether for a given model consensus is possible or not … and prove it!

Conensus #1: Shared Memory

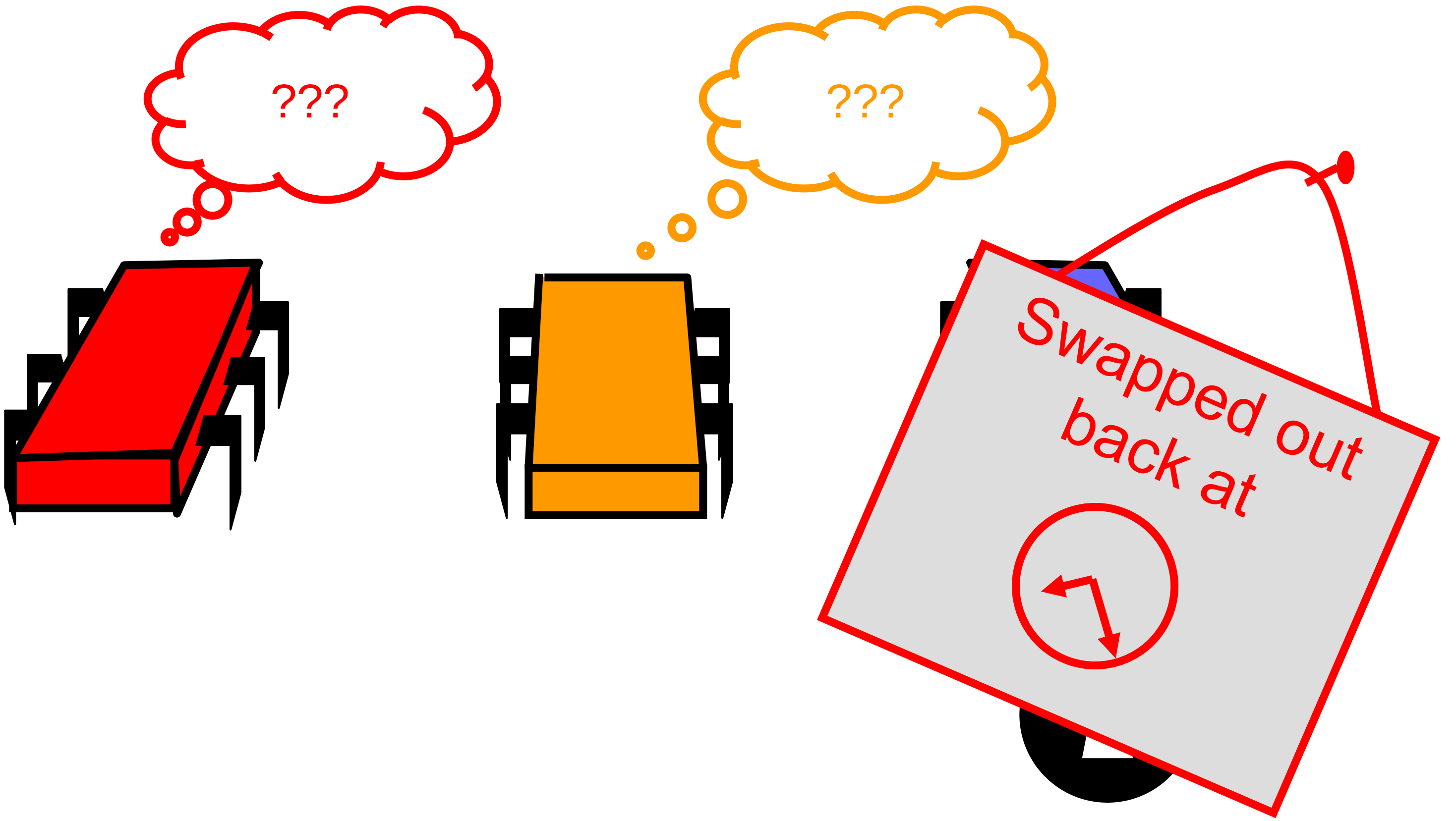
Problem:

- n processors, with $n > 1$
- Processors can atomically *read* or *write* (not both) a shared memory cell
- Must decide on one of the input values

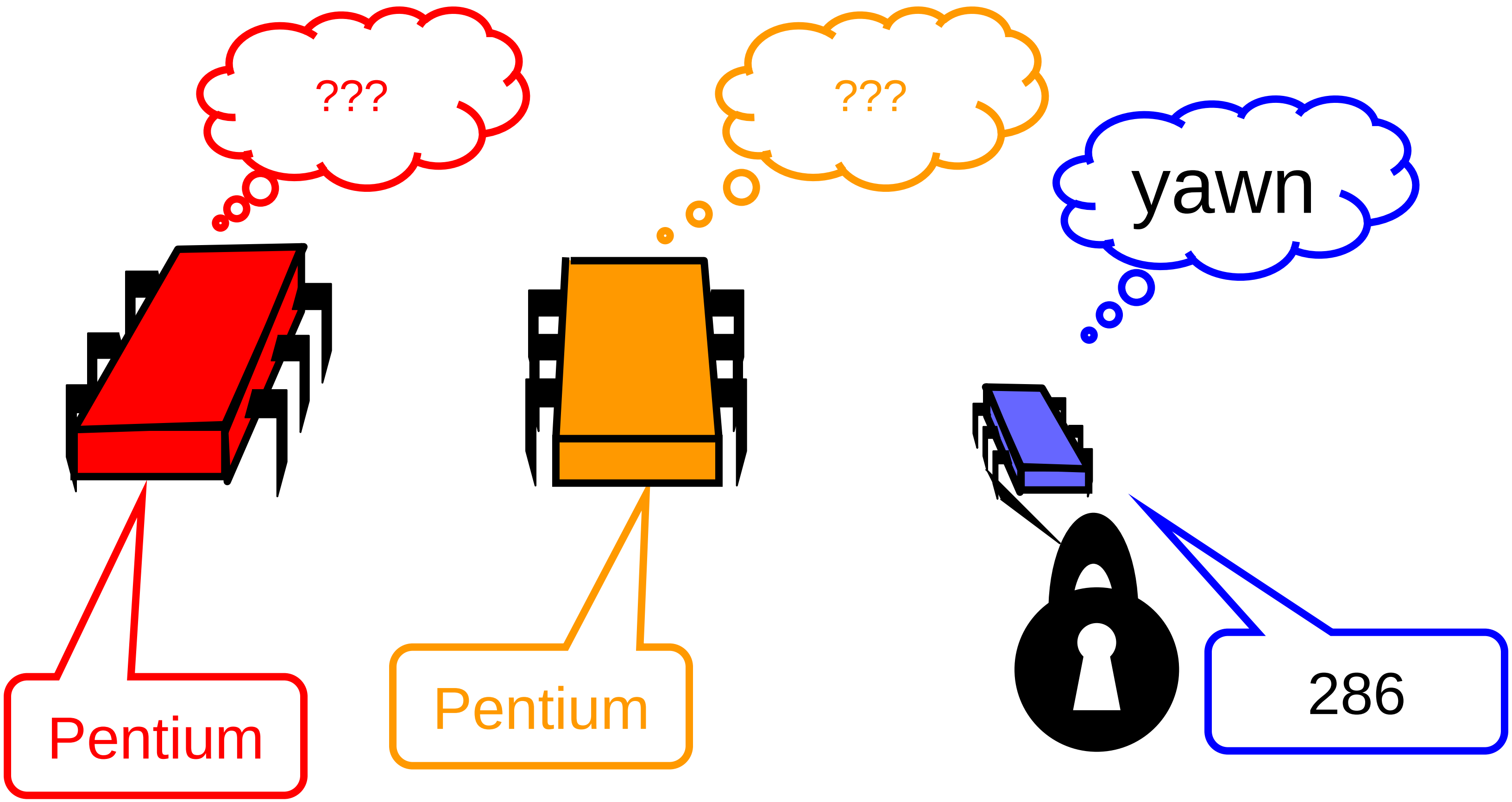
Idea:

- There is a designated memory cell c.
- Initially c is in a special state "?"
- Processor 1 writes its value v_1 into c, then decides on v_1.
- A processor j (j not 1) reads c until j reads something else than "?", and then decides on that.

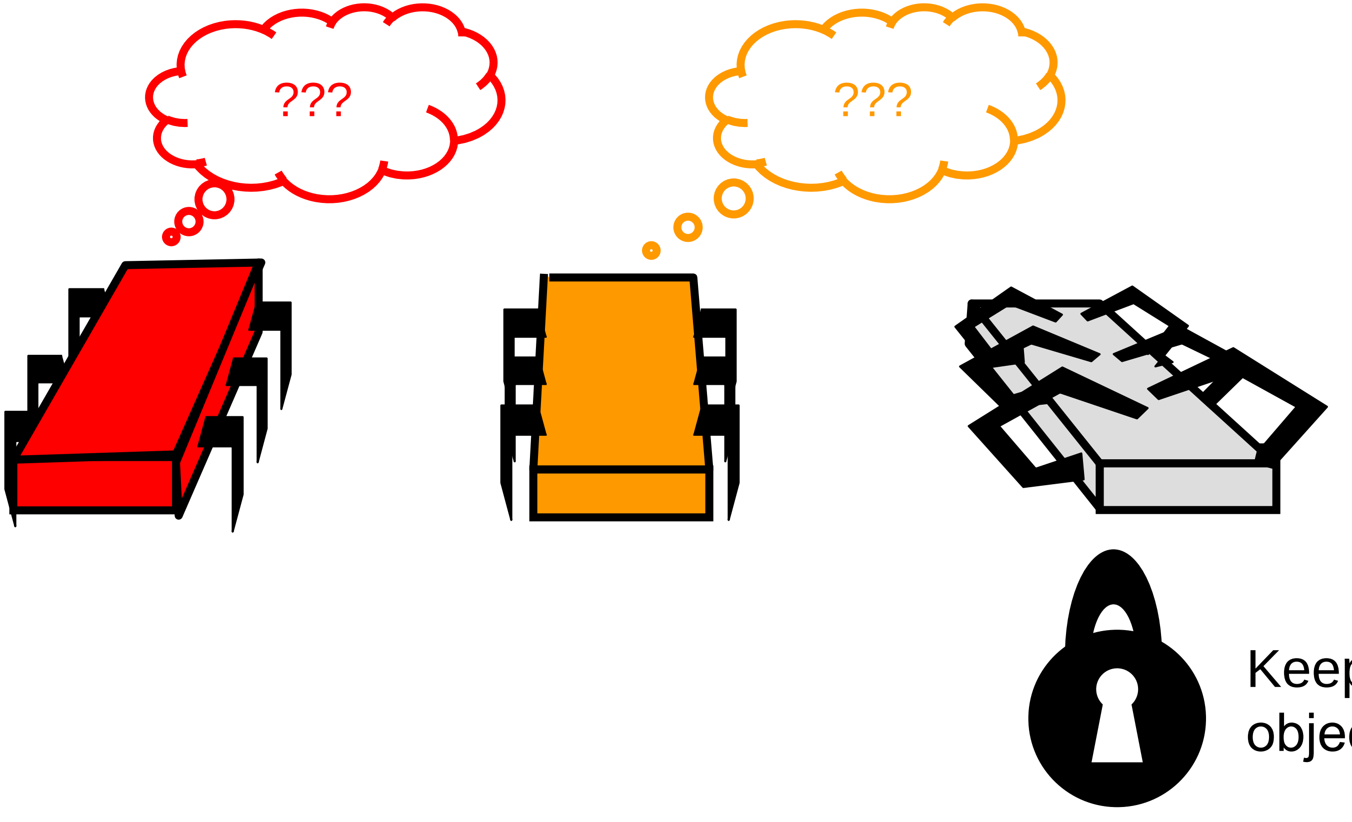
Problems: Unexpected Delay ...



Problems: ... Heterogenous Resources ...

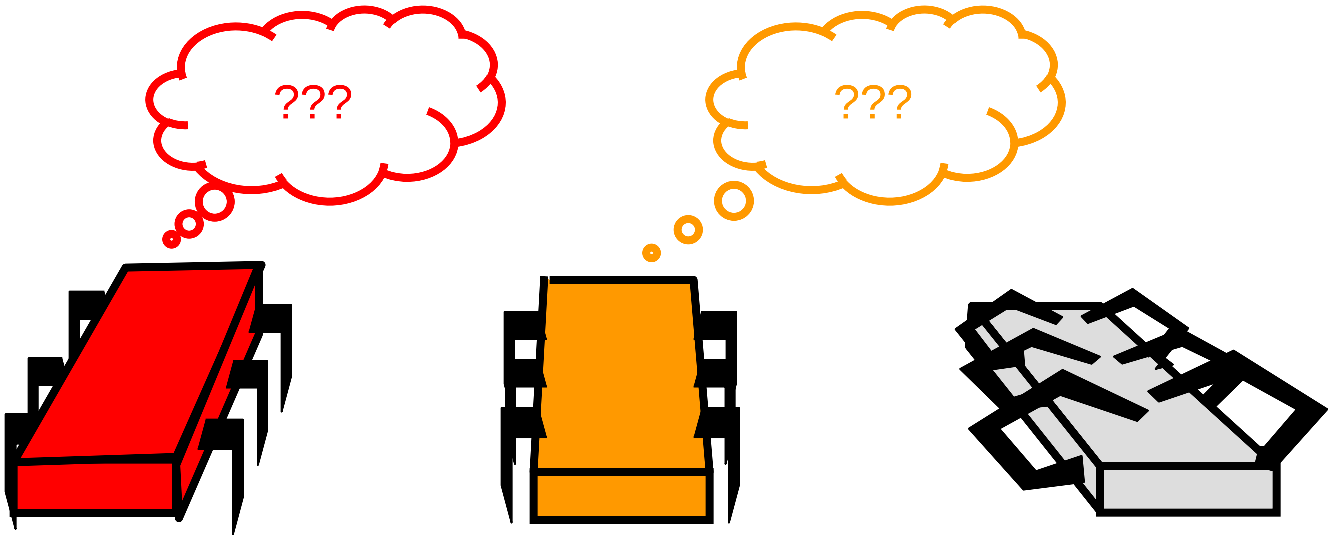


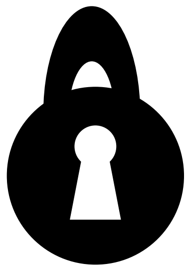
Problems: ... Fault-Tolerance?



Keeps lock on objects…

Problems: ... Fault-Tolerance?





Keeps lock on objects…

E.g., your algorithm:

when to decide on minimum? And minimum of which subset of processes?

Asynchronous! Has other process died??

Need to be “wait-free”!

Consensus #2: Wait-Free Shared Memory

- n processors, with n > 1
- Processors can atomically *read* or *write* (not both) a shared memory cell
- Processors might crash (halt)
- Wait-free implementation… huh?

Model

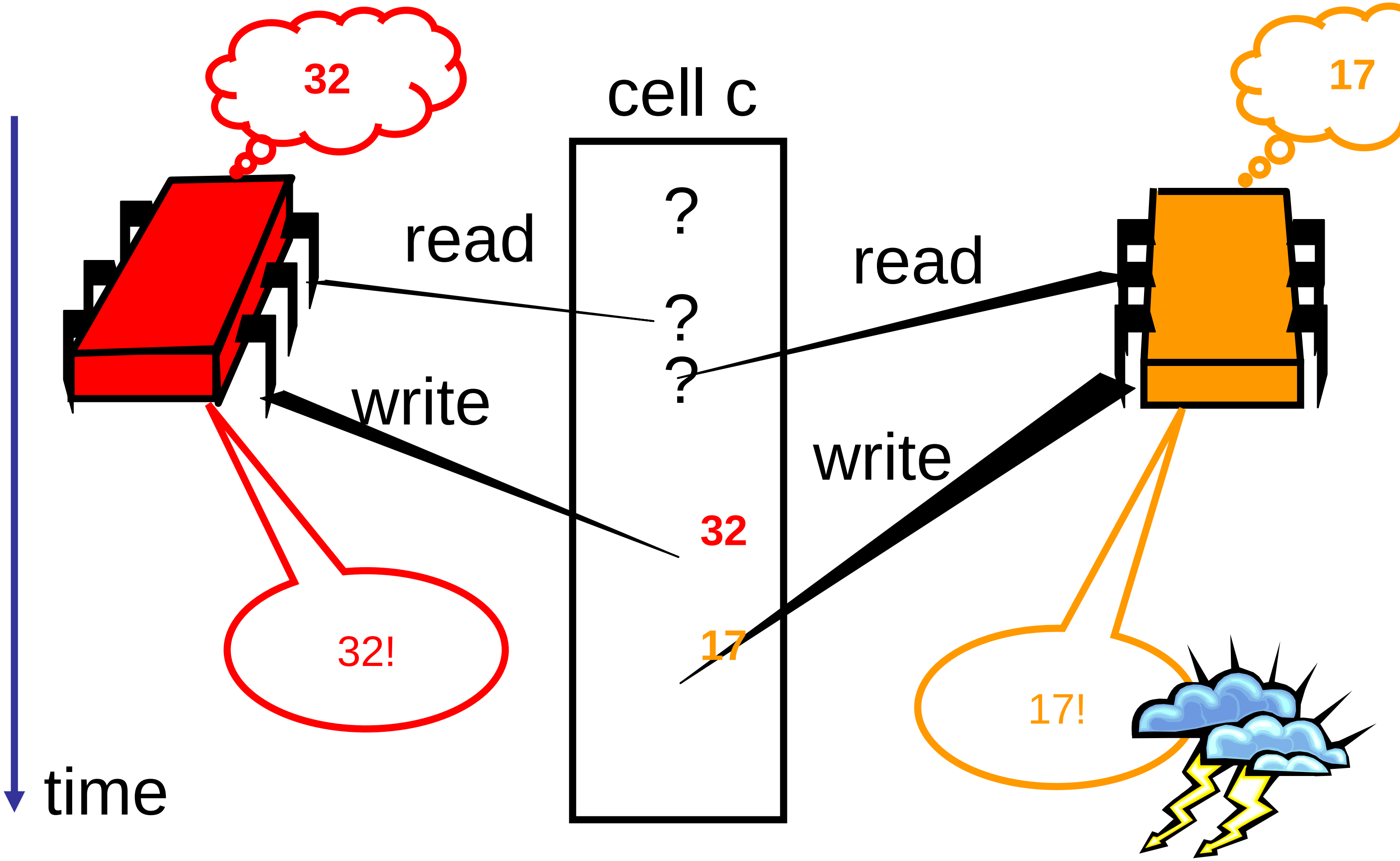
- Wait-free = every process (method call) completes in a finite number of its own steps
  - if scheduled sufficiently frequently, we're fine
- Register: object that supports read/write
  - not punctual, takes time!
- We assume that we have wait-free atomic register implementations
  - that is, it seems that reads and writes to same register do not overlap, and the real-time order is respected
  - real-time: response of previous operation precedes the invocation of the next operations
  - "linearizable"

A Correct, Wait-Free Algorithm?

- There is a cell c, initially c="?"
- Every processor i does the following

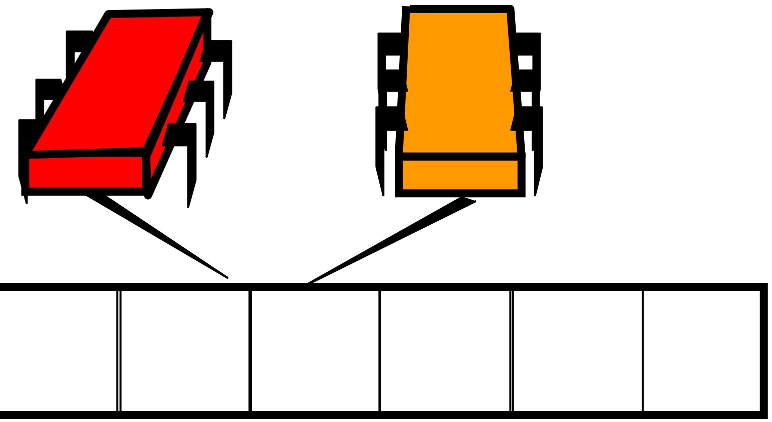
```
r = Read(c);
if (r == "?") then
        Write(c, v_i); decide v_i;
else
        decide r;
```

Correct?



Consensus #2 Impossible!

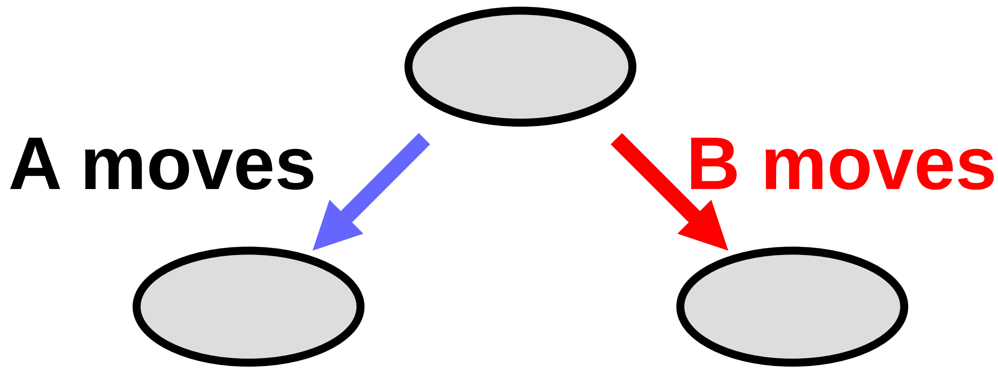
Theorem: Consensus #2 is impossible!



Proof Strategy:

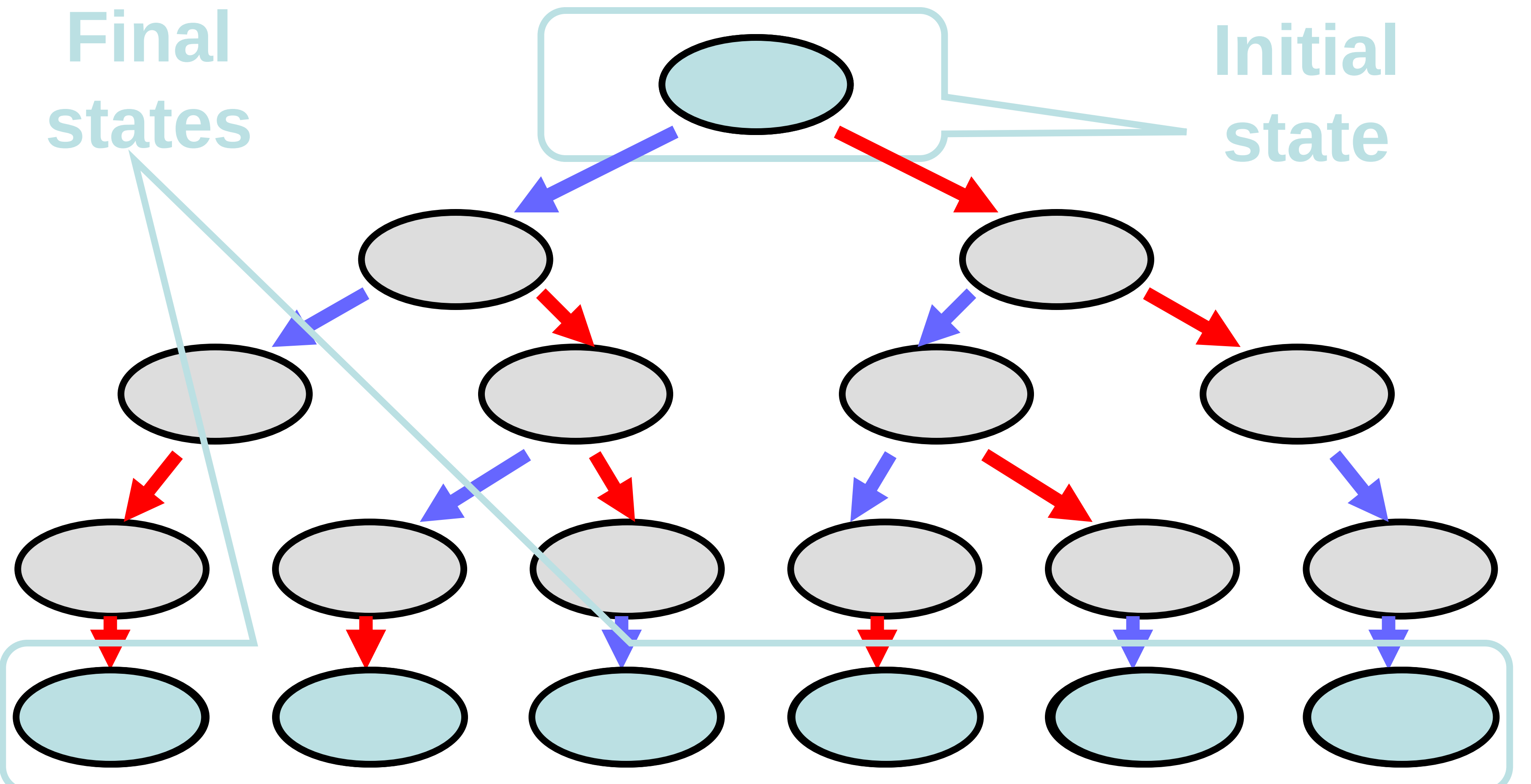
- Make it simple
  - n = 2, binary input
  - one or more r/w registers
- Assume that there is a protocol (choose yours!)
- Reason about the properties of any such protocol
- Derive a contradiction: choose “bad schedule”, i.e., who is next (asynchronous), who dies, …

Wait-Free Computation

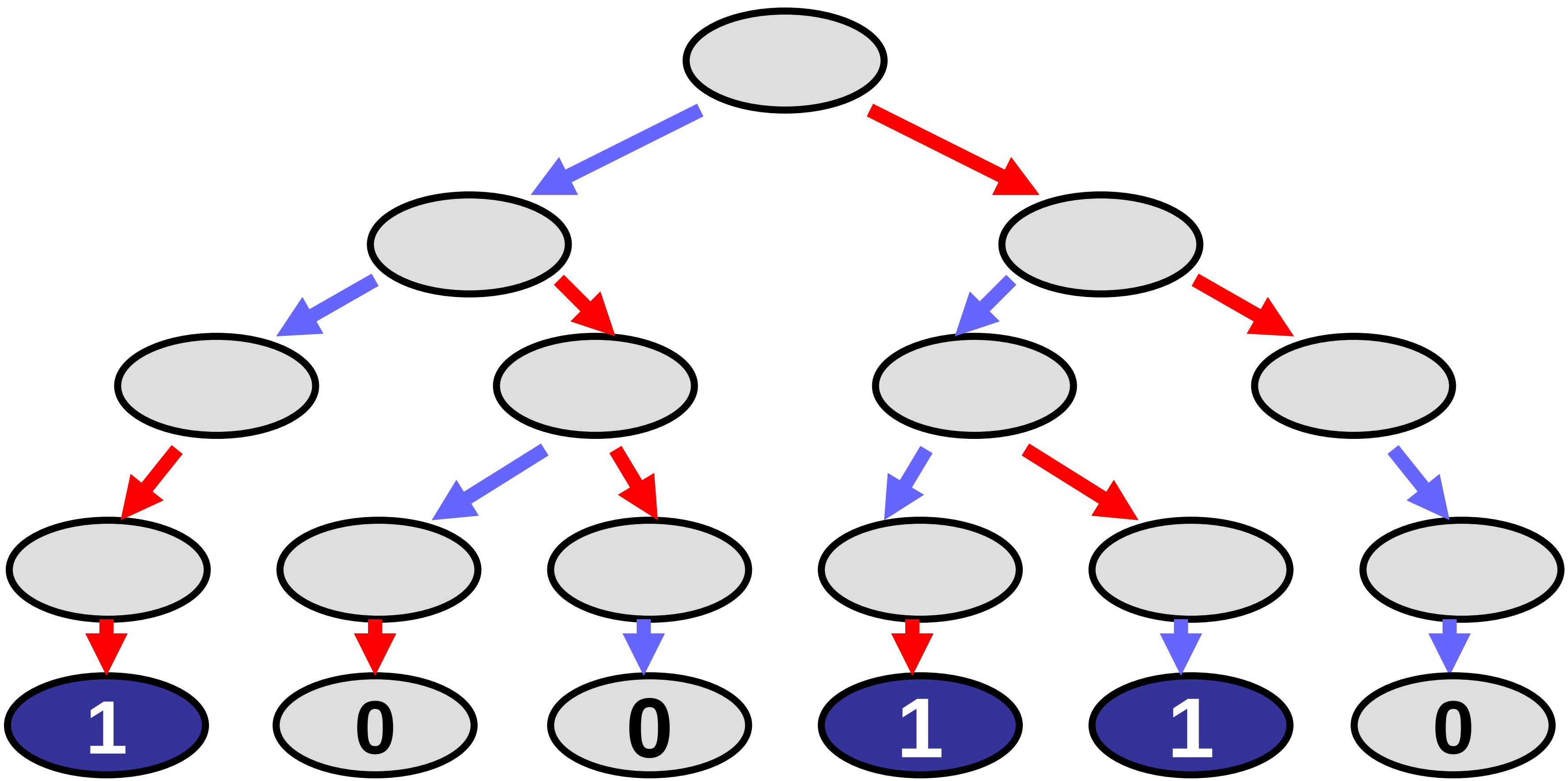


- Either A or B "moves" (atomic read/write!)
  - Asynchronous: "scheduler can choose!"
- Moving means
  - Register read
  - Register write

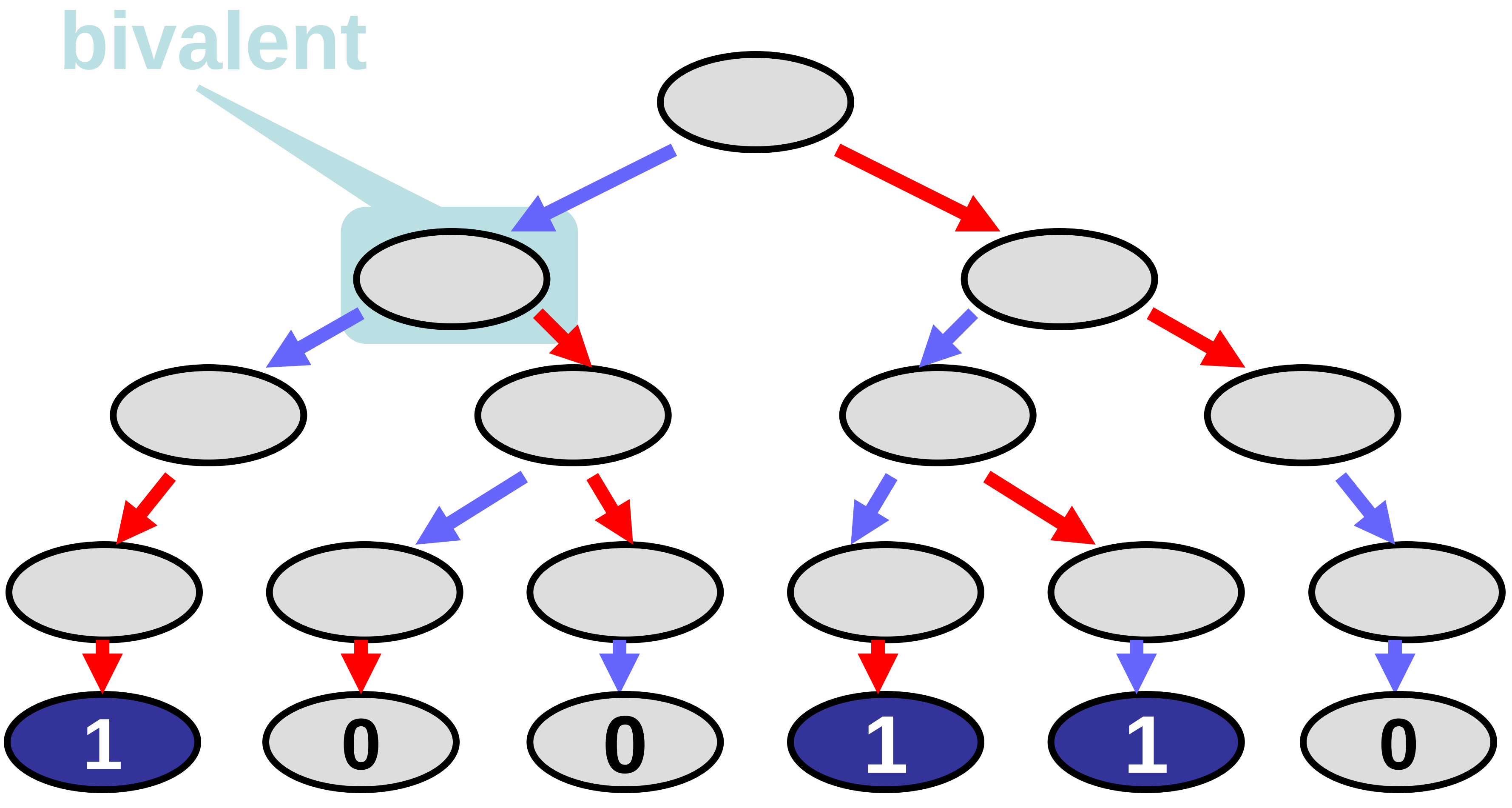
The 2-Move Tree



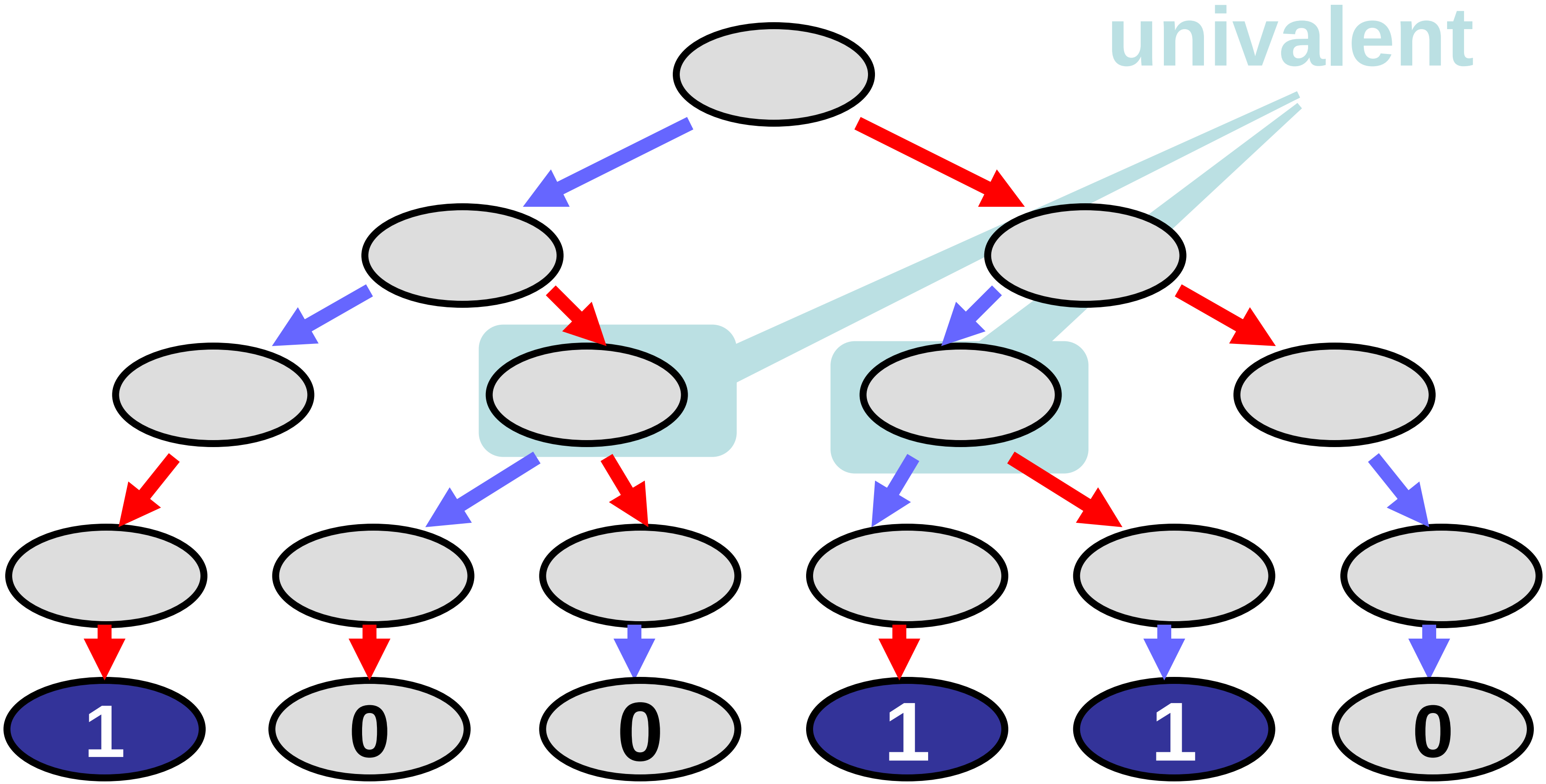
Decision Values



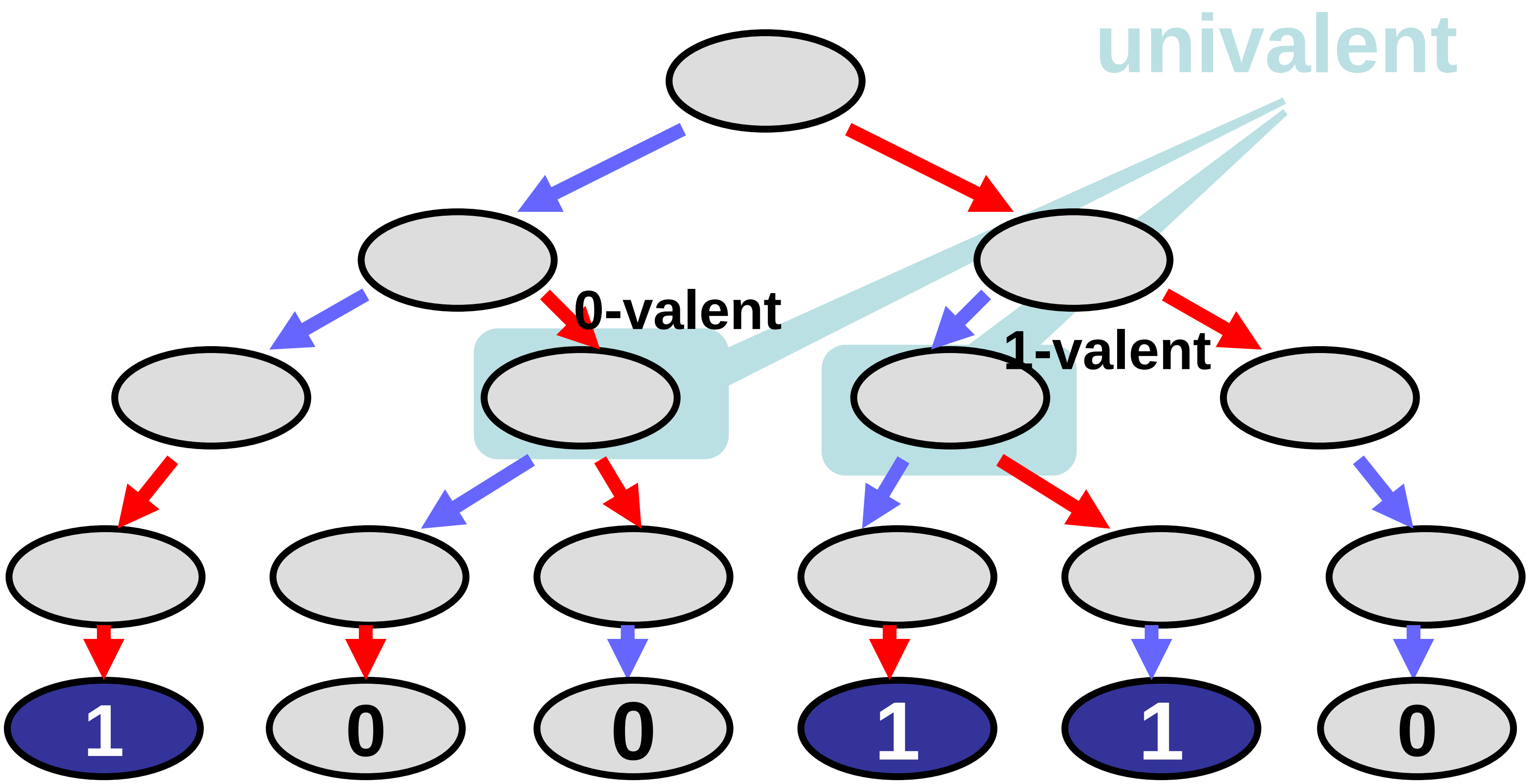
Bivalent: Both Possible



Univalent: Only One Possible



Univalent: Only One Possible



Summary

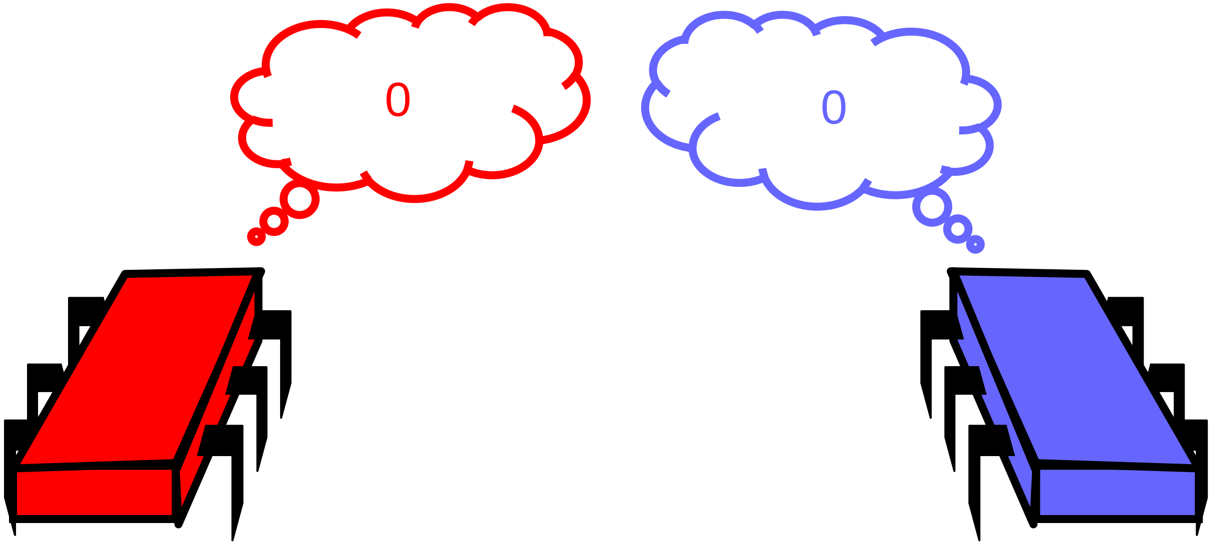
- Wait-free computation is a tree
- Bivalent system states
  - Outcome not fixed, even given the process inputs (but not the execution / who dies)
- Univalent states
  - Outcome is fixed
  - Maybe not “known” yet
  - 1-Valent and 0-Valent states

Claim

$\boxed{\text{Exists bivalent system state.}}$

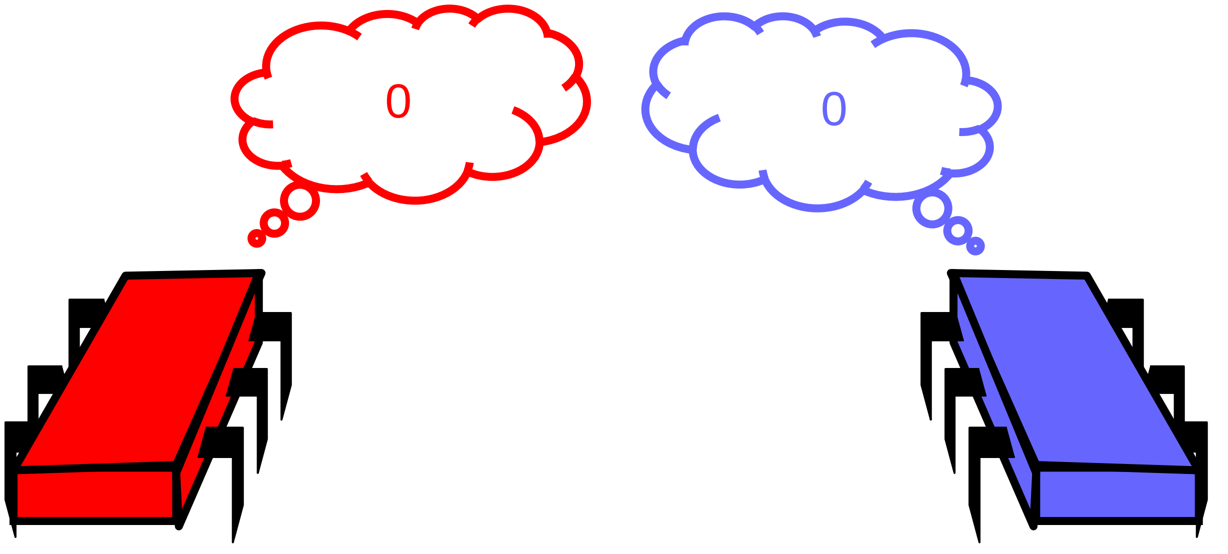
(The outcome is not always fixed from the start, even if process values are given.)

0-Valent Initial State



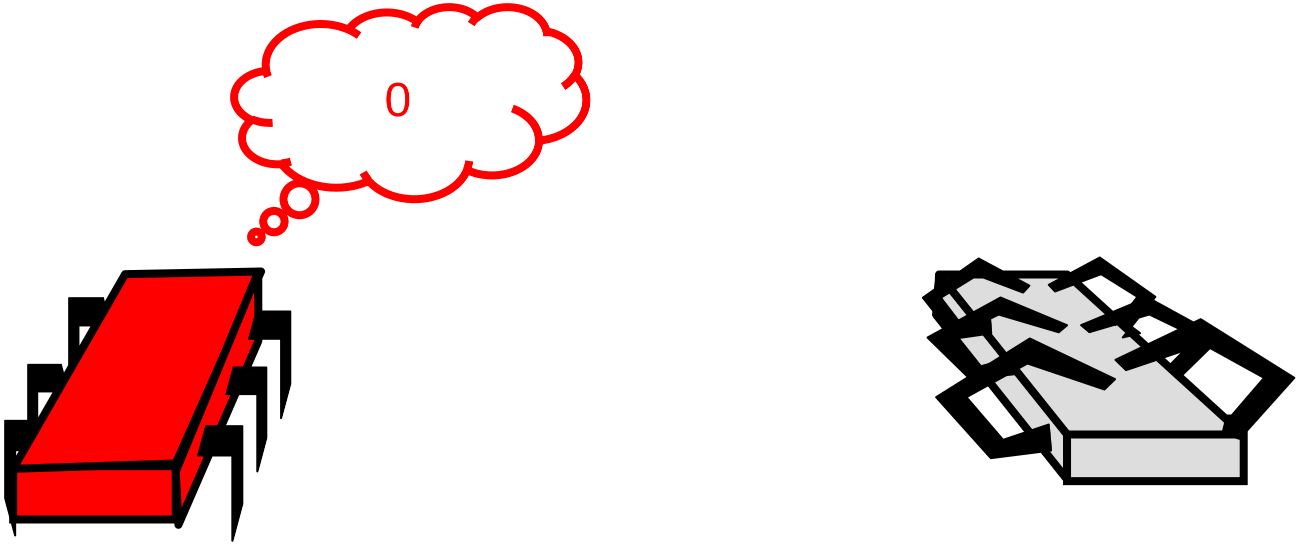
All executions lead to decision of 0

0-Valent Initial State



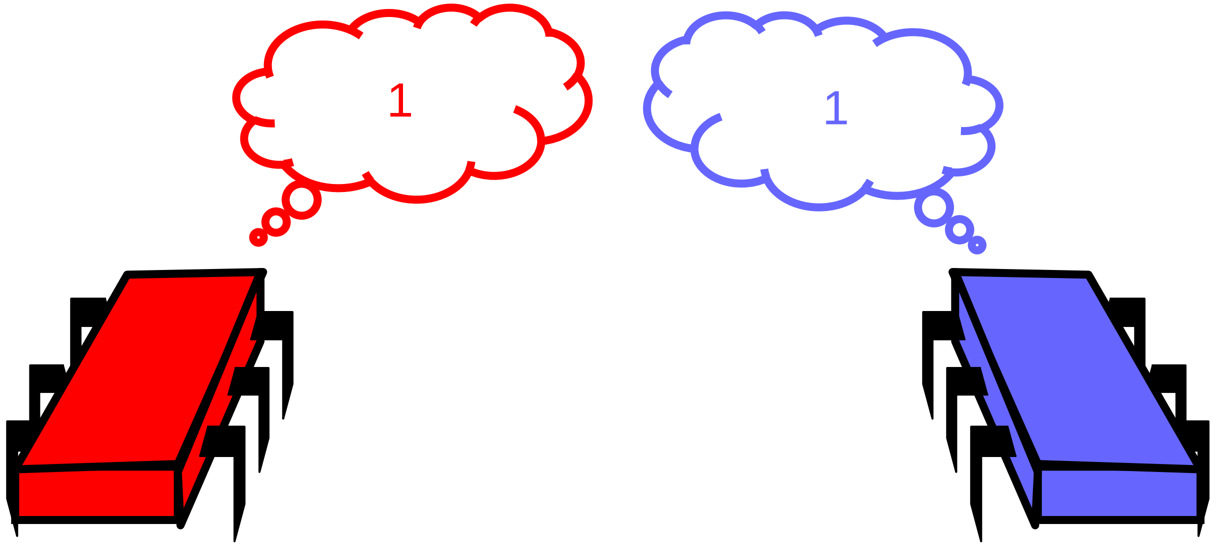
All executions lead to decision of 0

0-Valent Initial State



Solo execution by A also decides 0

1-Valent Initial State

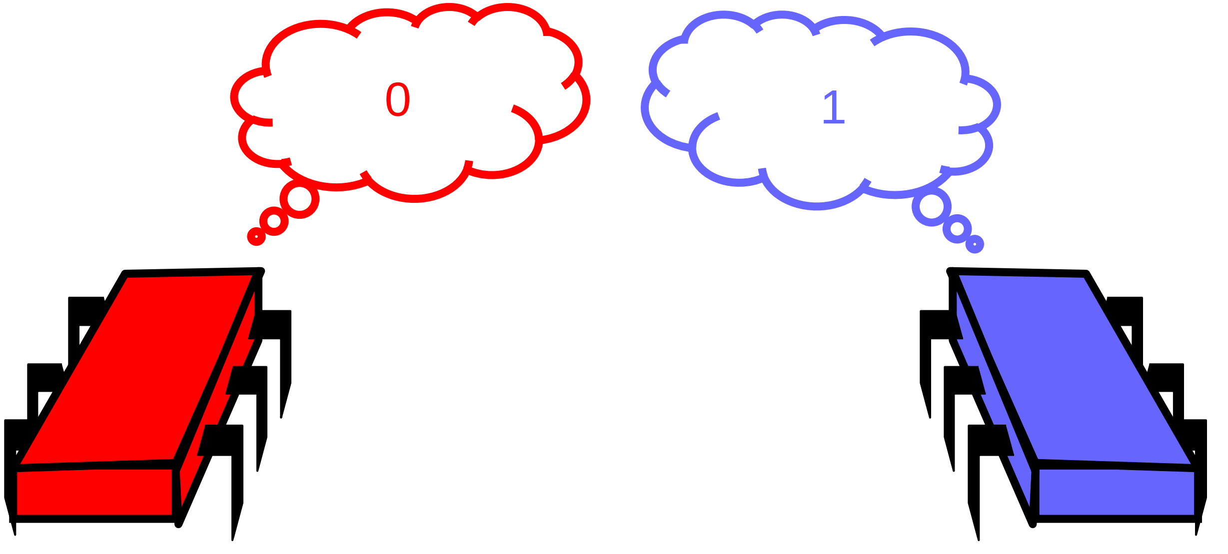


All executions lead to decision of 1

1-Valent Initial State

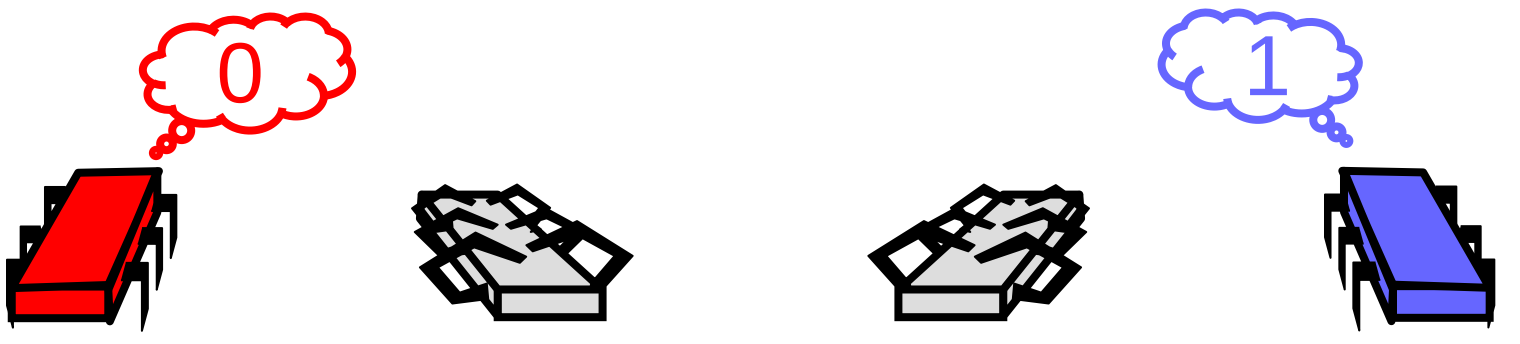
Solo execution by B also decides 1

Is Initial State Univalent?



Can all executions lead to the same decision? No, must depend on execution!

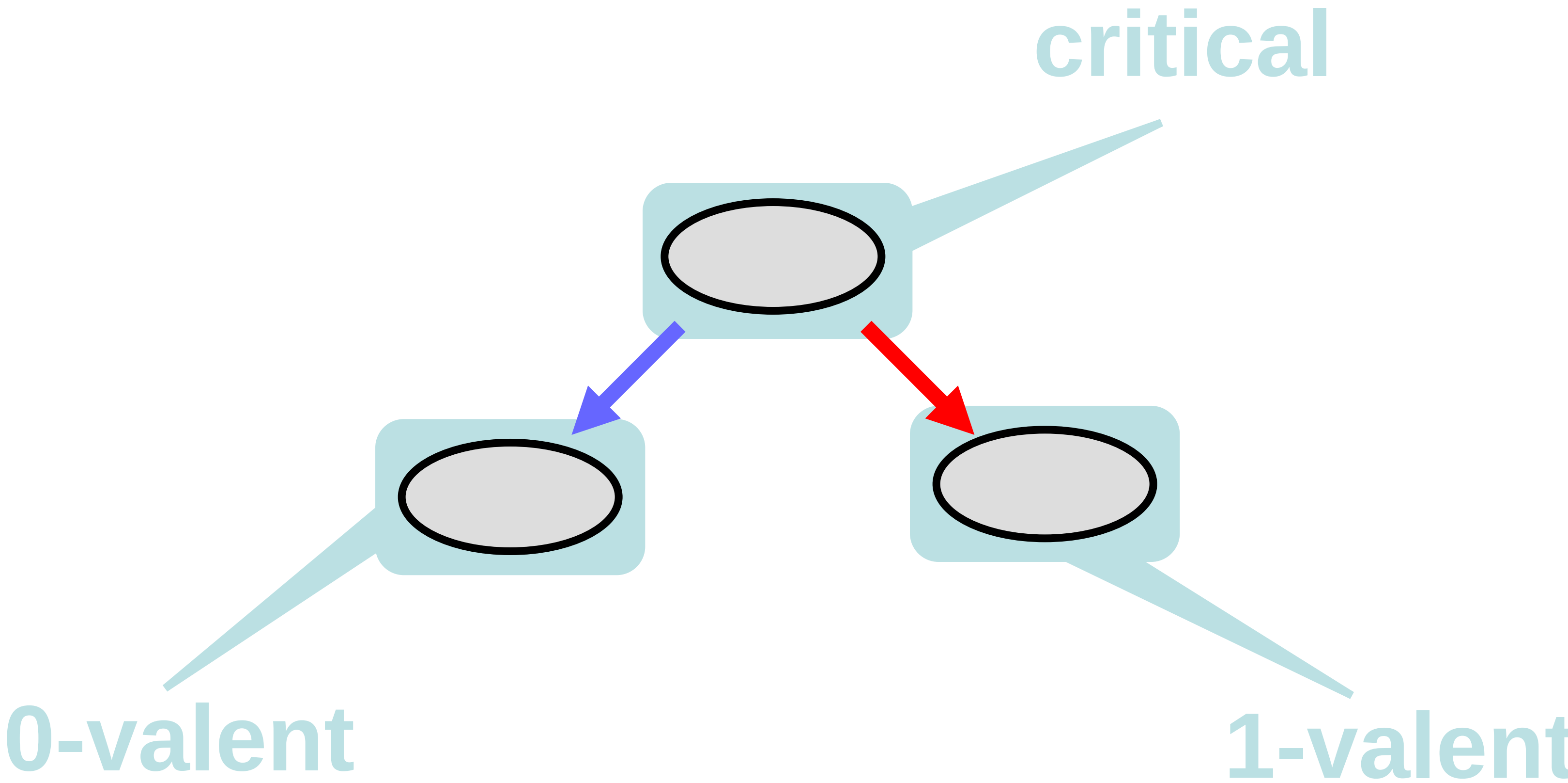
Bivalent Initial State



Solo execution by A must decide 0

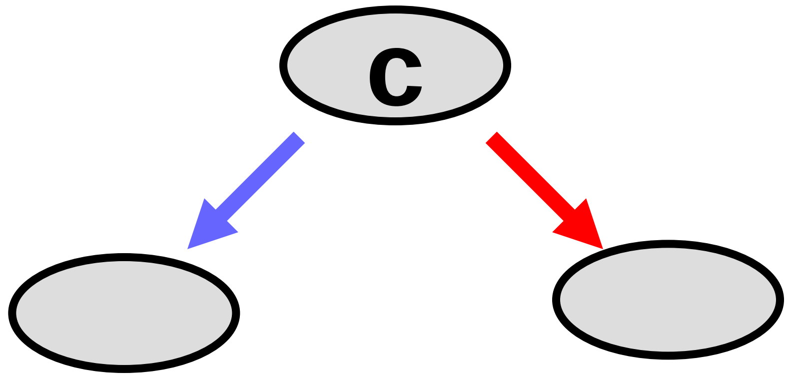
Solo execution by B must decide 1

Definition: Critical State



Critical States

- Starting from a bivalent initial state
- The protocol can reach a critical state
  - Otherwise we could stay bivalent forever
  - And the protocol is not wait-free
  - Note: if only left 0-valent, right still bivalent: not critical yet, take c lower



0-valent

1-valent

If A goes first, protocol decides 0

If B goes first, protocol decides 1

Critical States

- Starting from a bivalent initial state
- The protocol can reach a critical state
  - Otherwise we could stay bivalent forever
  - And the protocol is not wait-free
  - N

We will show that processes cannot distinguish who goes first: Contradiction to critical state!

0-valent

1-valent

If A goes first, protocol decides 0

If B goes first, protocol decides 1

Model Dependency

- So far, memory-independent!
- True for
  - Registers
  - Message-passing
  - Carrier pigeons
  - Any kind of asynchronous computation

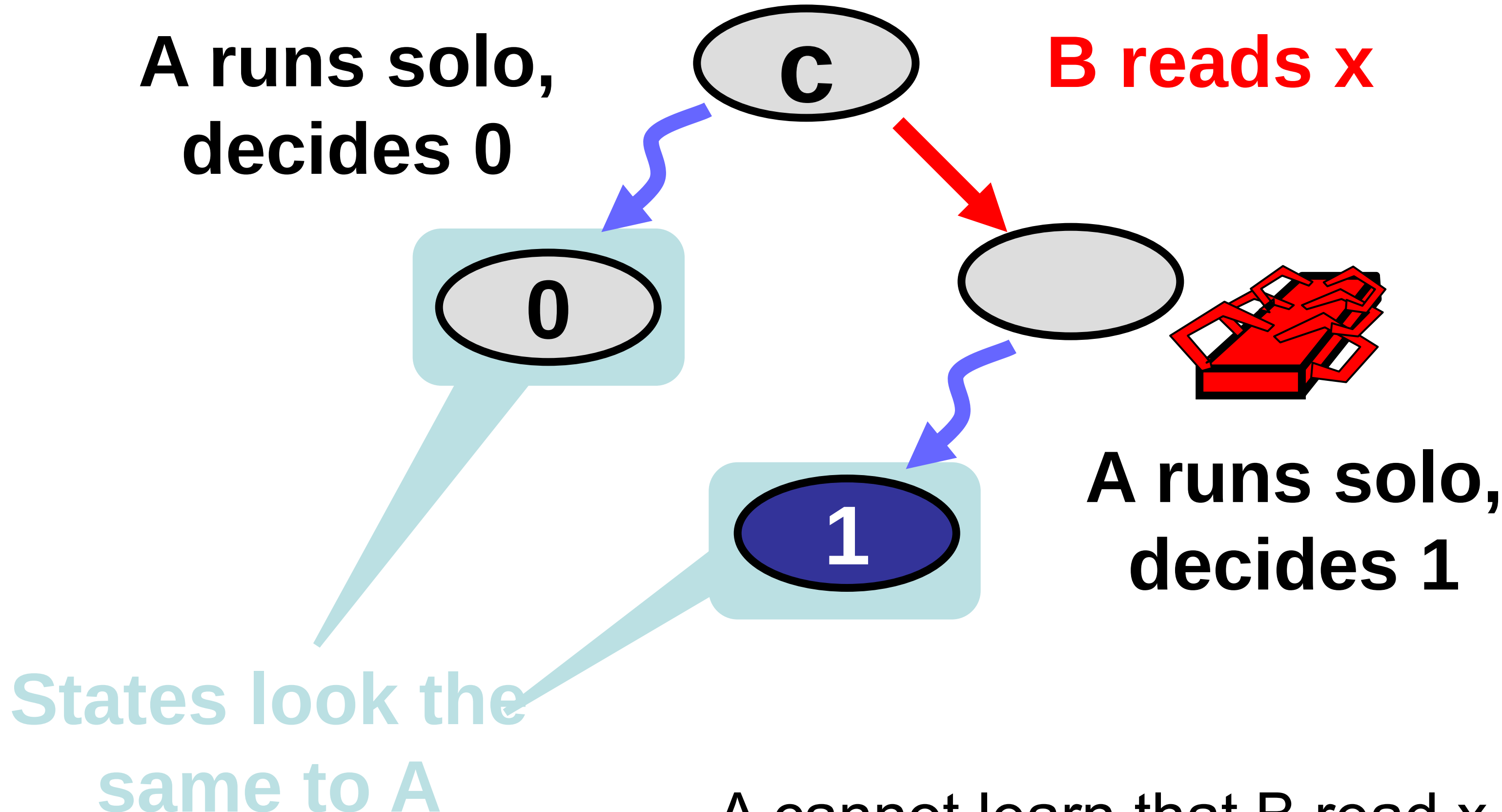
What Are the Threads Doing?

- Reads and/or writes
- To same/different registers (one or more registers!)

Possible Interactions after critical state:

	x.read()	y.read()	x.write()	y.write()
x.read()	?	?	?	?
y.read()	?	?	?	?
x.write()	?	?	?	?
y.write()	?	?	?	?

Reading Registers: Interaction of «Read»??

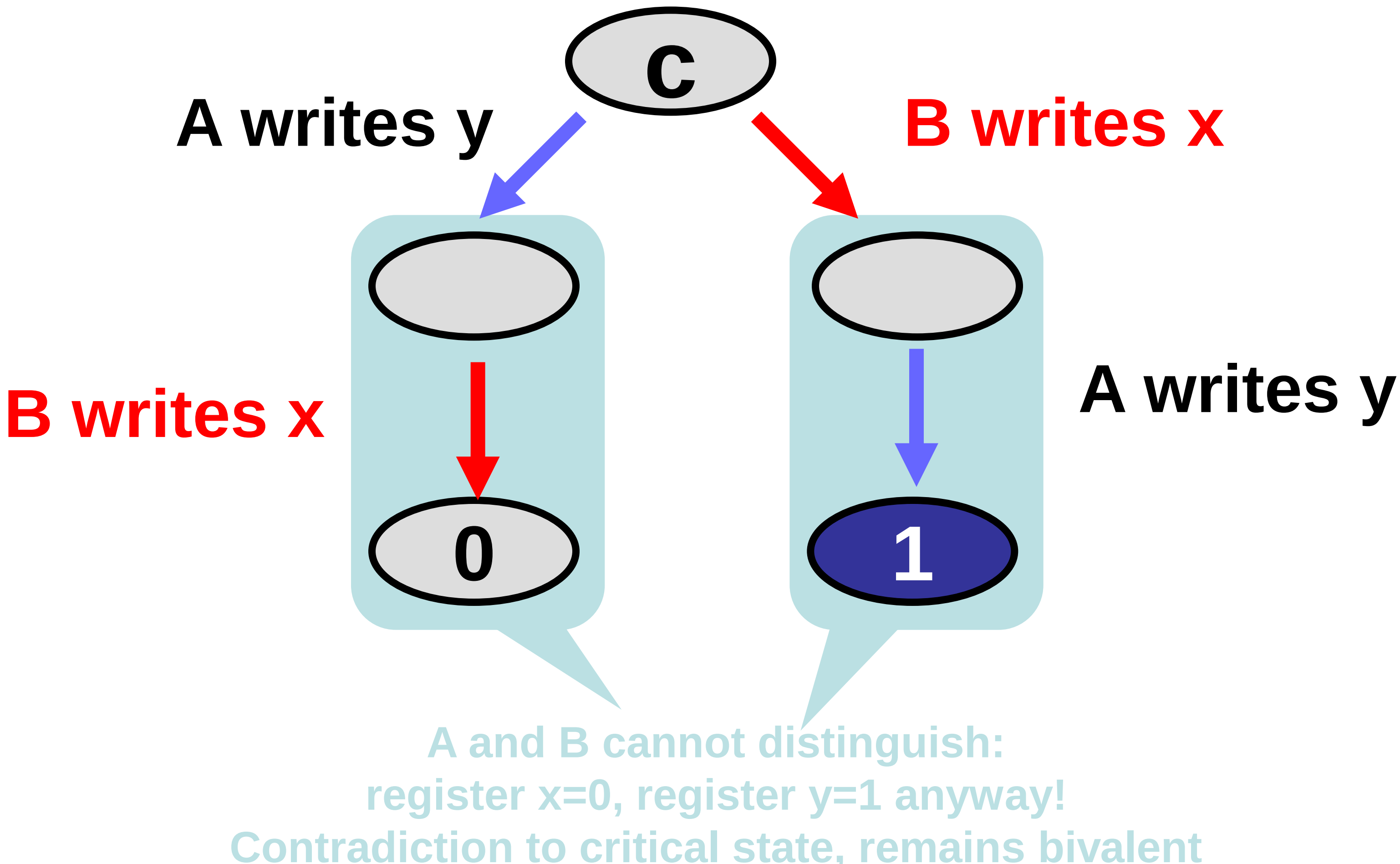


A cannot learn that B read x…

Possible Interactions

	x.read()	y.read()	x.write()	y.write()
x.read()	no	no	no	no
y.read()	no	no	no	no
x.write()	no	no	?	?
y.write()	no	no	?	?

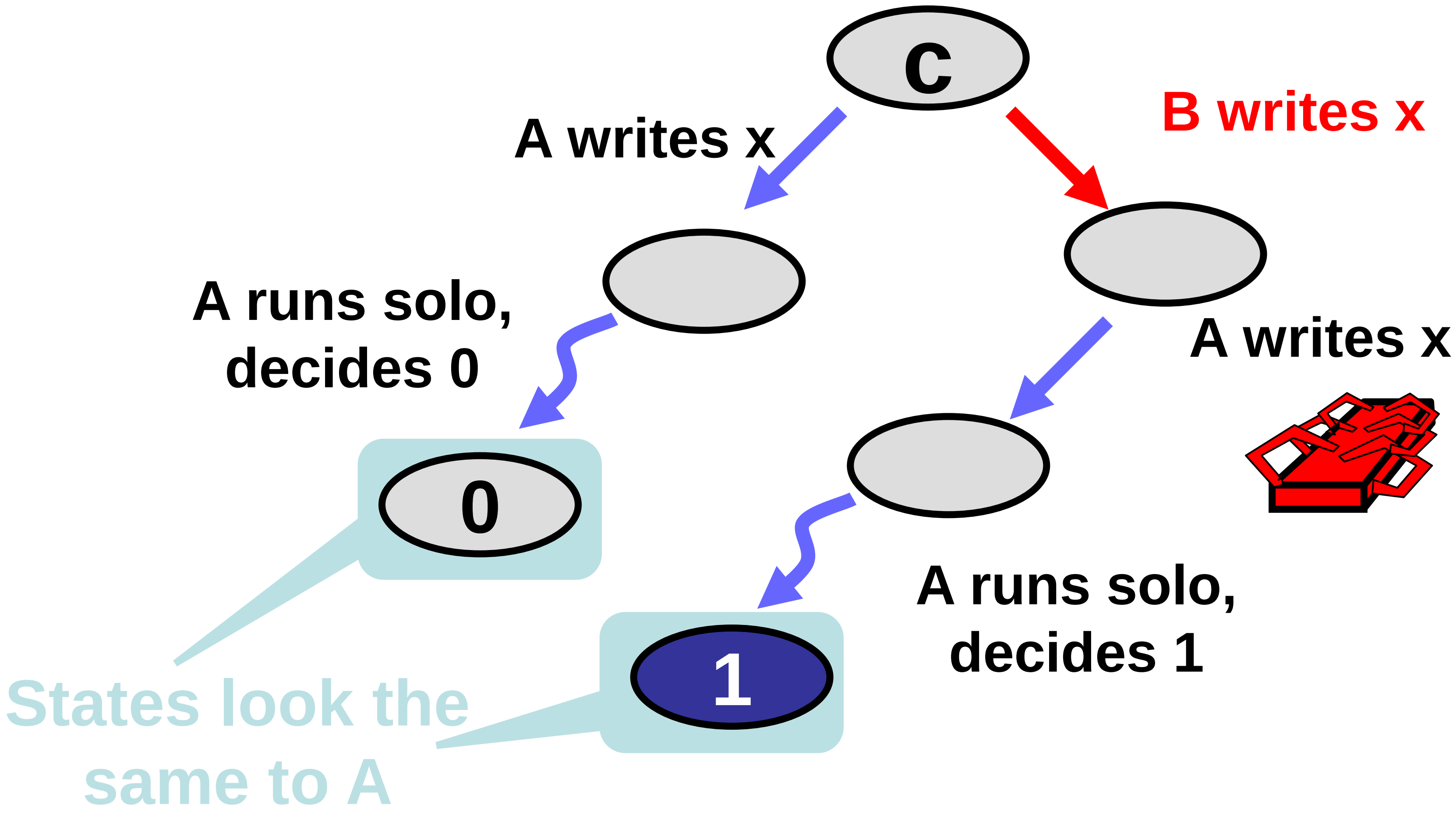
Writing Distinct Registers: Write-Write Different Registers



Possible Interactions

	x.read()	y.read()	x.write()	y.write()
x.read()	no	no	no	no
y.read()	no	no	no	no
x.write()	no	no	?	no
y.write()	no	no	no	?

Writing Same Registers: Write-Write Same Register



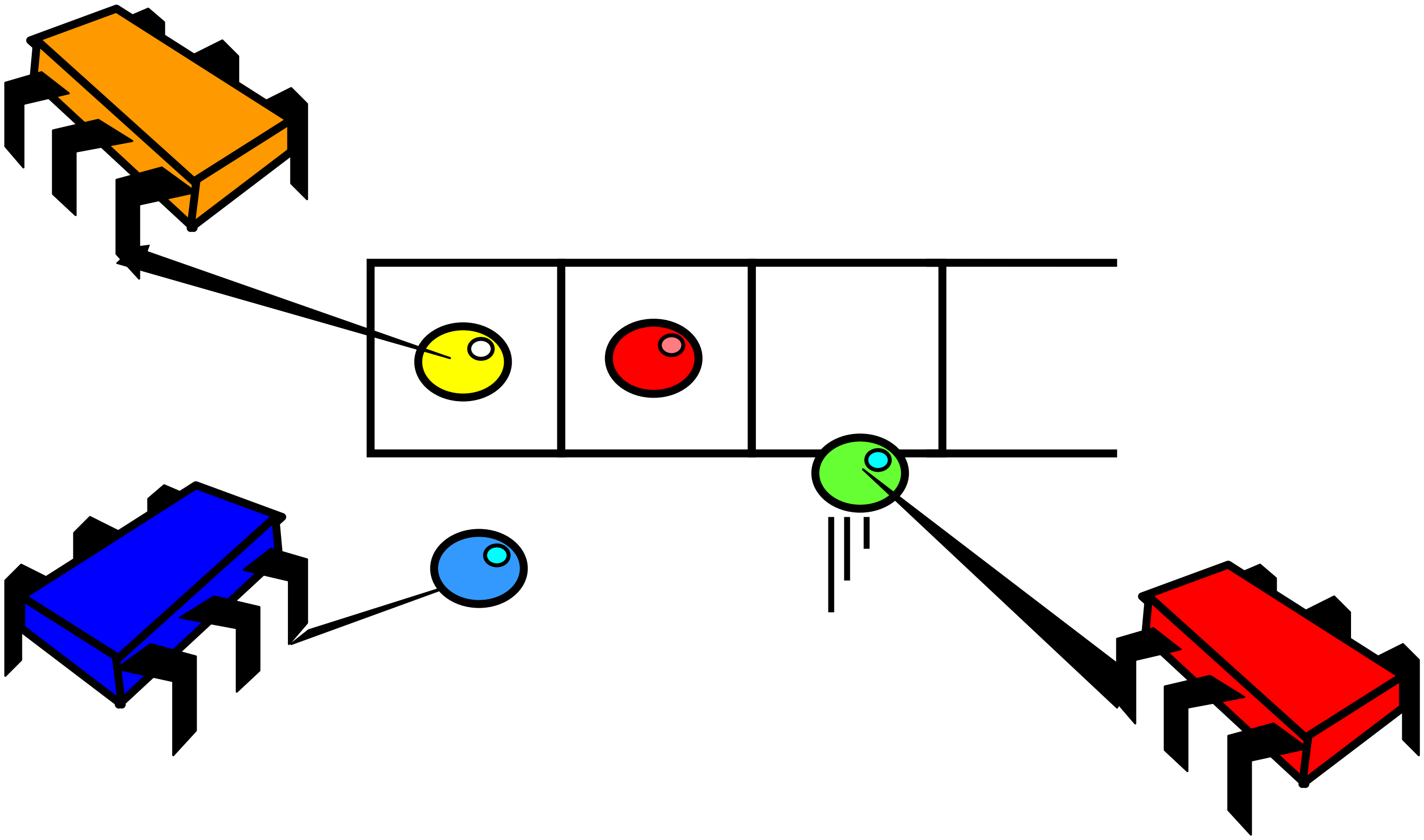
That's all, folks!

	x.read()	y.read()	x.write()	y.write()
x.read()	no	no	no	no
y.read()	no	no	no	no
x.write()	no	no	no	no
y.write()	no	no	no	no

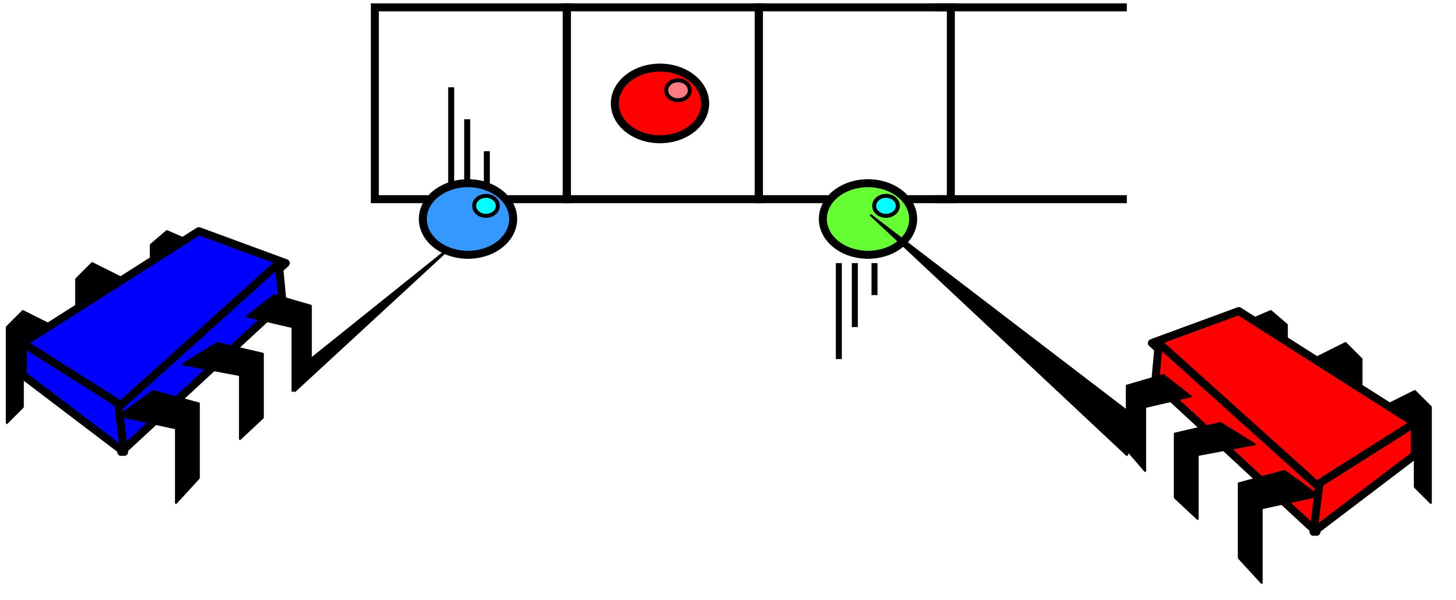
Theorem

- It is impossible to solve consensus using read/write atomic registers
  – Assume protocol exists
  – It has a bivalent initial state
  – Must be able to reach a critical state
  – Case analysis of interactions
    - Reads vs others
    - Writes vs writes

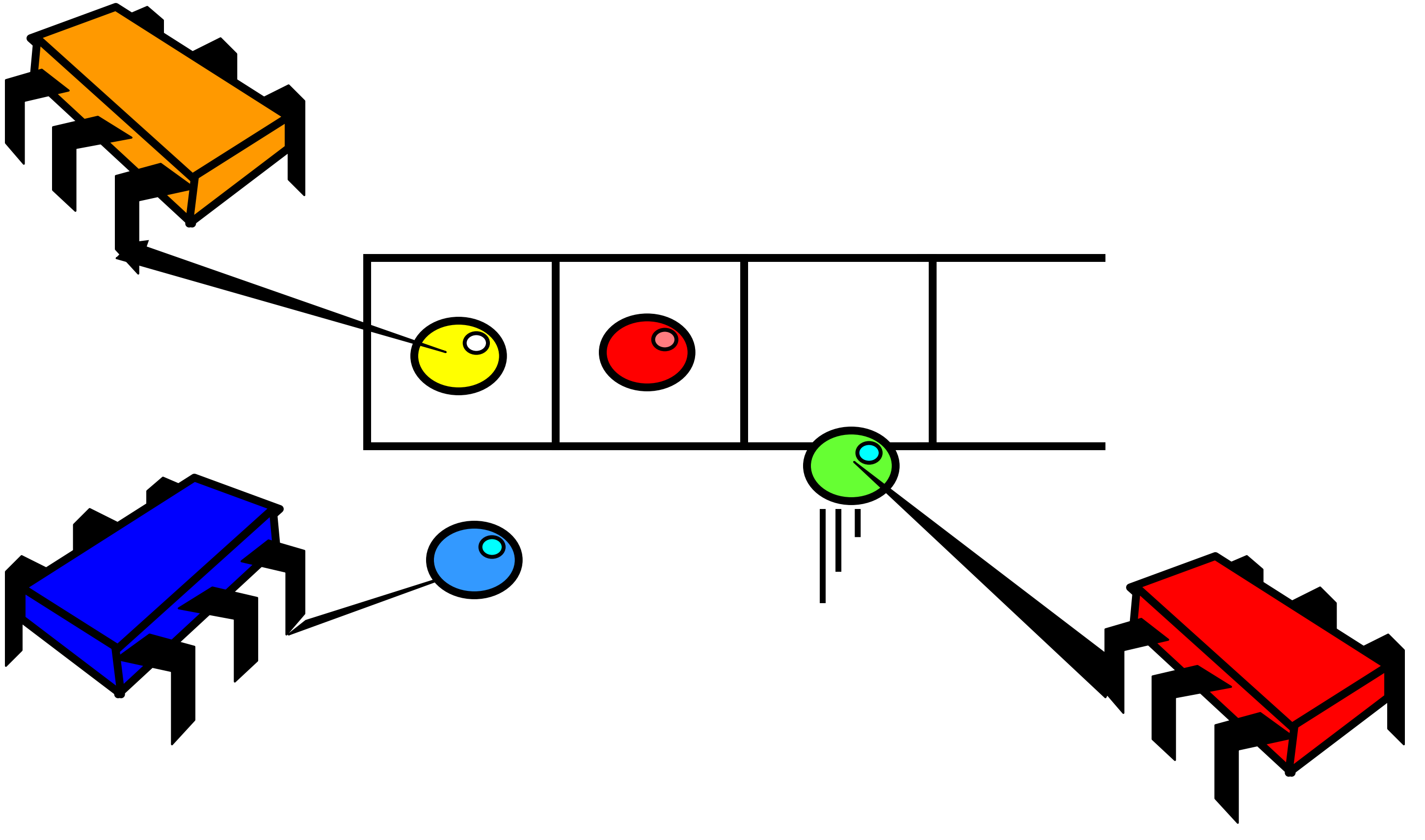
What does Consensus have to do with Distributed Systems?

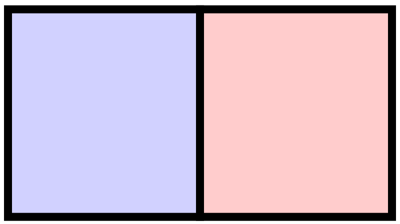


We want to build a concurrent FIFO queue...

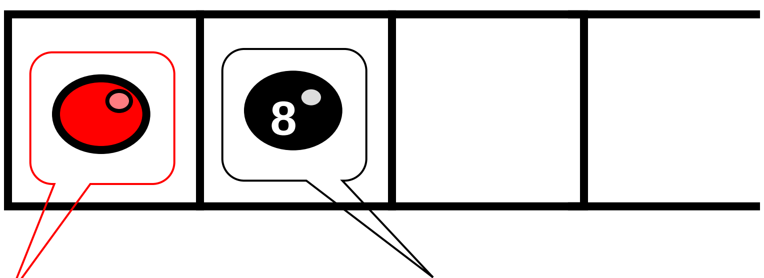


... with multiple dequeuers.





2-element array

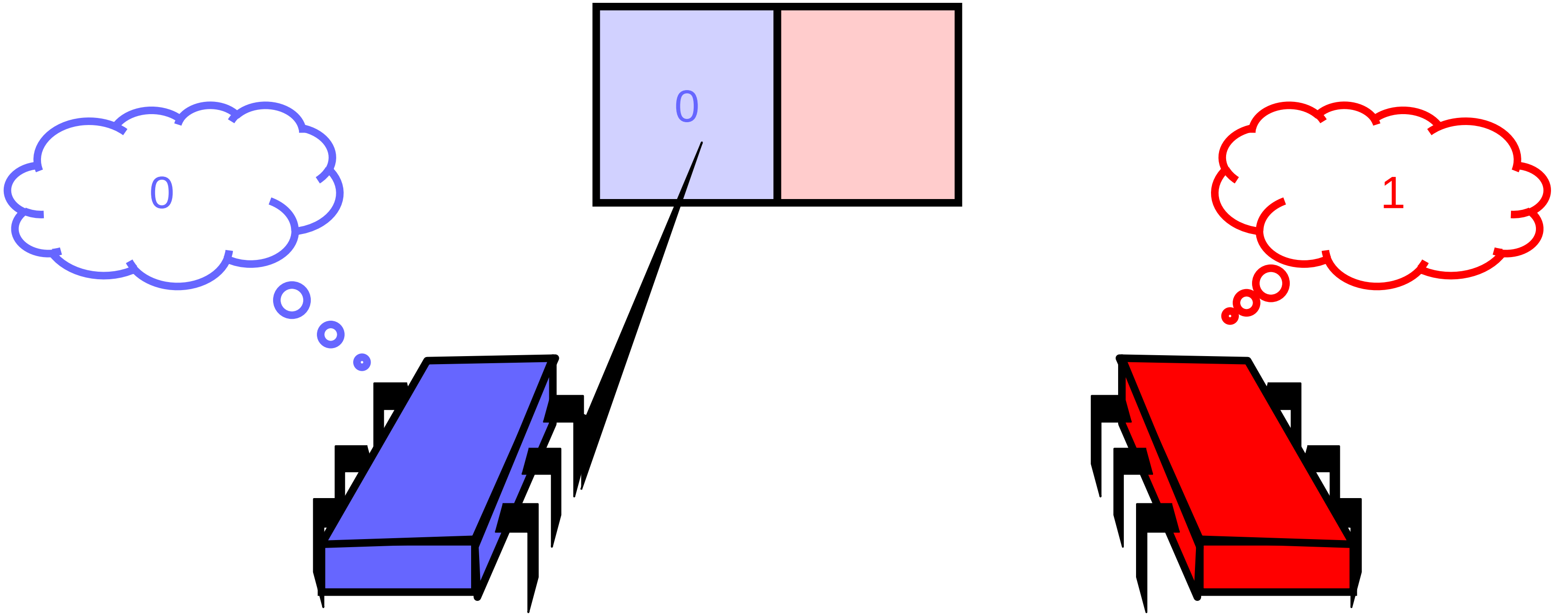


FIFO Queue with red and black balls

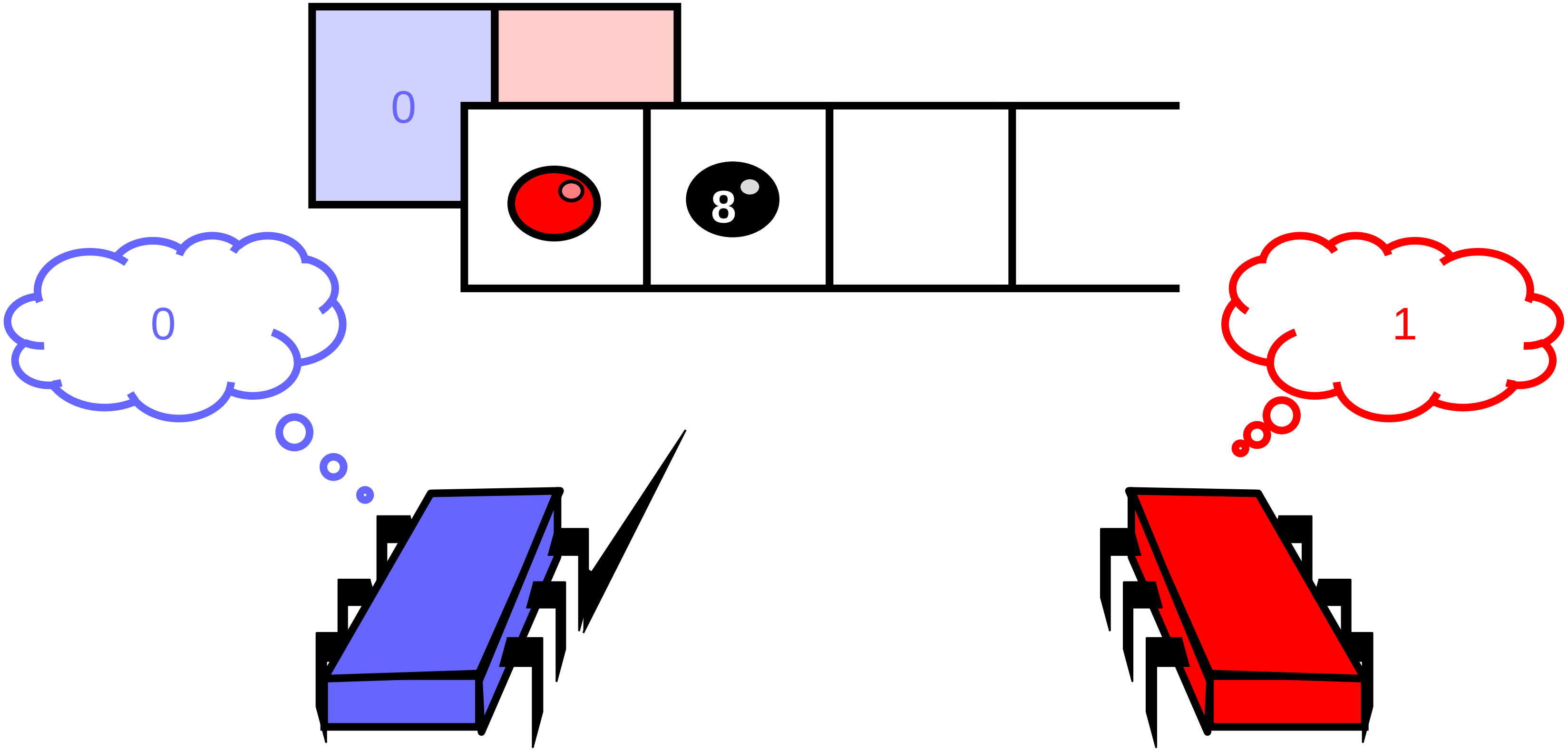
Rough Protocol Idea

1. Put my value to “my” register
2. Dequeue initialized queue: am I winner or loser?

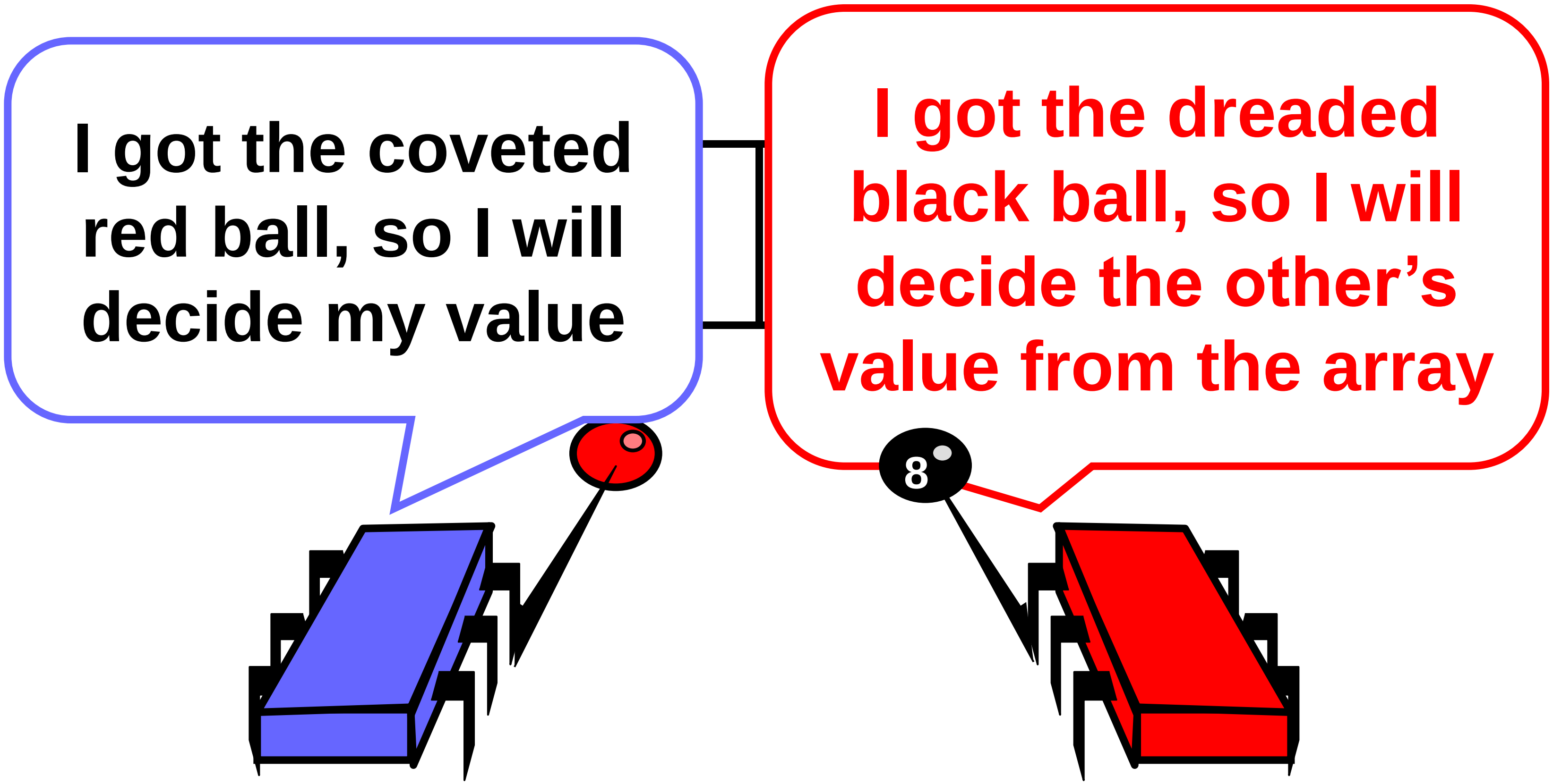
Write Value to Array



Protocol: Take Next Item from the Queue



The Protocol



Why does it work?

- If one thread gets the red ball
- Then the other gets the black ball
- Winner can take her own value
- Loser can find winner’s value in array
  - Because threads write array before dequeuing from queue