

Network Algorithms: Exercise 5

Tree algorithms



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1 BFS Tree (20 + 10 = 30 points)

Recall the Bellman-Ford like algorithm to construct a BFS in an asynchronous environment, and assume that the underlying network is a hypercube of dimension 3.

- How many message transmissions are necessary in the worst-case? Please give a real number (not asymptotically) and explain why this is indeed a bad or even worst case!
- Did you use FIFO links in a)? Explain why this can make a difference.

2 License to Match (30 + 10 = 40 points)

In preparation of a highly dangerous mission, the participating agents of the gargantuan Liechtensteinian secret service (LSS) need to work in pairs of two for safety reasons. All members in the LSS are organized in a tree hierarchy. Communication is only possible via the official channel: an agent has a secure phone line to his direct superior and a secure phone line to each of his direct subordinates. Initially, each agent knows whether or not he is taking part in this mission. The goal is for each agent to find a partner.

- Devise an algorithm that will match up a participating agent with another participating agent given the constrained communication scenario. A “match” consists of an agent knowing the identity of his partner and the path in the hierarchy connecting them. Assume that there is an even number of participating agents so that each one is guaranteed a partner. Furthermore, observe that¹ the phone links connecting two paired-up agents need to remain open at all times. Therefore, you cannot use the same link (i.e., an edge) twice when connecting agents with their partners.
- What are the time and message (i.e., “phone call”) complexities of your algorithm?

3 License to Distribute (30 + 10 = 40 points)

We consider another day at the office of the LSS as in Exercise 2. After the above mission was successful, the involved agents collected a large number of sensitive documents. Some agents might have a lot of them and others have none. Now they need to distribute the documents throughout the agency such that each person in the LSS has the same amount of data to process.

- Assume that there are n agents in the LSS and that there is also a total of n documents. Devise a way for the agents to distribute their sensitive data: In the end, each agent should have exactly one document. The communication scheme is the same as above.
- How good is your algorithm with respect to time and number of messages? You may assume that arbitrarily many documents can be sent in a single message.

¹in the case of an emergency where they lose contact