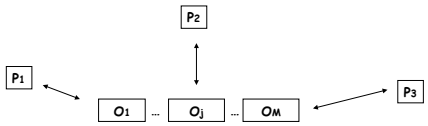
FDS 2012:
Shared memory basics

Shared memory model

- Processes communicate by applying operations on and receiving responses from *shared objects*
- A shared object is a state machine
  - ✓States
  - ✓Operations/Responses
  - ✓Sequential specification
- Examples: read-write registers, T&S,C&S,LLSC,…



Read-write register

- Stores *values* (in a value set V)
- Exports two operations: read and write
  - ✓Write takes an argument in V and returns ok
  - ✓Read takes no arguments and returns a value in V

We assume *well-formed* executions:

A process never invokes an operation before its previous invocation returns

Shared memory guarantees

Processes invoke operations on the shared objects and:

- Liveness: the operations eventually return *something*
- Safety: the operations never return *anything incorrect*

Liveness

- An operation is *complete* if its invocation is followed by a matching response
  - ✓write(v) -> ok
  - ✓read() -> a value in V
- A process invoking an operation may fail (stop taking steps) before receiving a response
- A process is correct (in a given run) if it never fails

Under which condition a correct process makes progress?

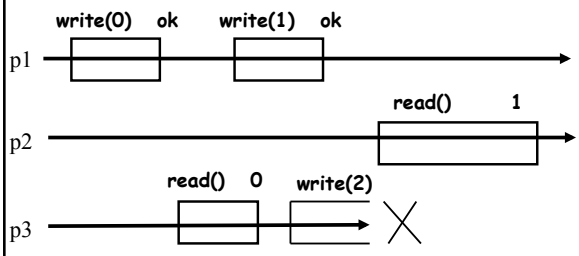
Wait-freedom: unconditional progress

Every operation invoked by a correct process eventually completes

All objects considered in this class are wait-free

We consider well-formed runs: a process never invokes an operation before returning from the previous invocation

A shared memory run



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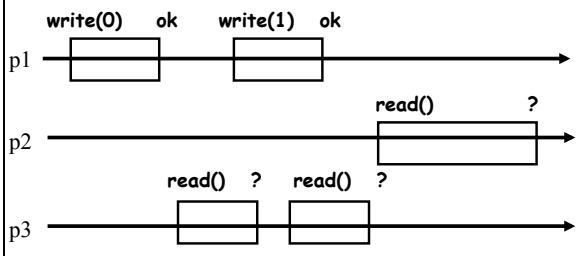
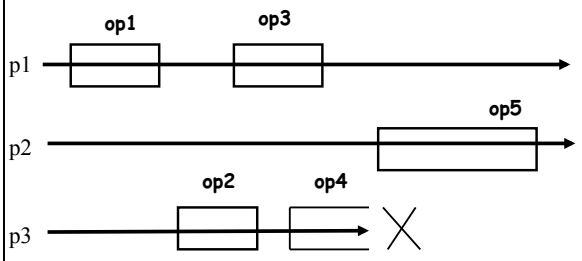


Operation precedence

- Operation op1 precedes operation op2 in a run R if the response of op1 precedes (in global time) the invocation of op2 in R
- If neither op1 precedes op2 nor op2 precedes op1 than op1 and op2 are concurrent

Operation precedence



Safety (registers)

Informally, every read operation returns the "last" written value (the argument of the "last" write operation)

- ✓What does the "last" mean?
- ✓What if operations overlap?

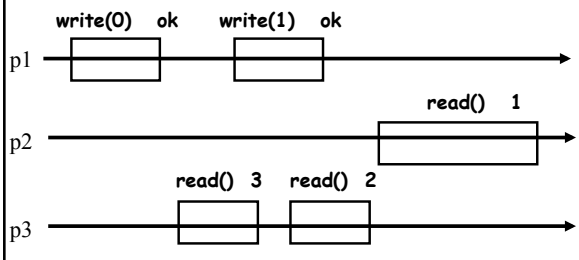
Safety criteria

- Safe registers: every read that does not overlap with a write returns the last written value
- Regular registers: every read returns the last written value, or the concurrently written value

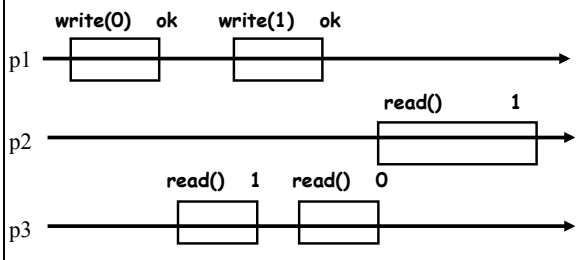
  (assuming one writer)

- Atomic registers: the operations can be totally ordered, preserving legality and precedence (linearizability)
  - ✓≈ if read1 returns v, read2 returns v', and read1 precedes read2, then write(v') cannot precede write(v)

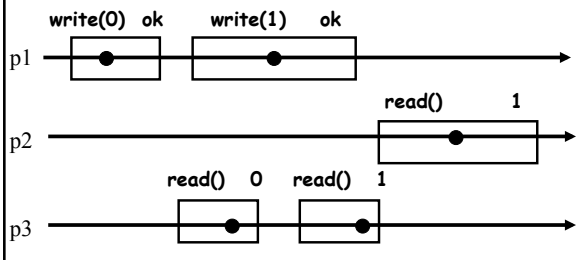
Safe register



Regular register



Atomic register



Space of registers

- Values: from binary (V={0,1}) to multi-valued
- Number of readers and writers: from 1-writer 1-reader (1W1R) to multi-writer multi-reader (NWNR)
- Safety criteria: from safe to atomic

1W1R binary safe registers can be used to implement an NWNR multi-valued atomic registers!

Transformations

From 1W1R binary safe to 1WNR multi-valued atomic

I. From safe to regular (1W1R)
II. From one-reader to multiple-reader (regular binary or multi-valued)
III. From binary to multi-valued (1WNR regular)
IV. From regular to atomic (1W1R)
V. From 1W1R to 1WNR (multi-valued atomic)

1WNR binary safe -> 1WNR binary regular

Let p1 be the only writer and 0 be the initial value

Code for process p1:

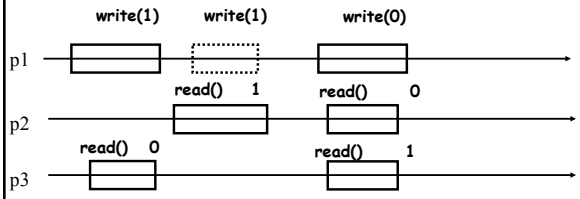
```
initially:
        shared 1WNR safe register R := 0
        lv := 0        \\ last written value

upon write(v)
        if v ≠ lv then
            lv := v
            R.write(v)
        return ok

upon read()
        return R.read()
```

1WNR binary safe -> 1WNR binary regular

- Correctness:
  - ✓R is touched only to change its value
  - ✓ both 0 and 1 are legal values in case of concurrency!



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1W1R (binary regular) -> 1WNR (binary regular)

Let p1 be the only writer and 0 be the initial value

Code for process pi:

```
initially:
        shared array R[1..N] of 1W1R binary regular registers := 0^N
        // For all i, R[i] is written by p1 and read by pi

upon read()
        return R[i].read()

upon write(v)  // if i=1
        for all j do R[j].write(v)
        return ok
```


1W1R (binary regular) -> 1WNR (binary regular)

- Correctness:
  - ✓enough to consider a read that does not overlap with any write
  - ✓the last written value cannot be missed
- Works also for multi-valued and safe registers

Is it also atomic?

Transformations

From 1W1R binary safe to 1WNR multi-valued atomic

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II. From one-reader to multiple-reader (regular binary or multi-valued)
III. From binary to multi-valued (1WNR regular)
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V. From 1W1R to 1WNR (multi-valued atomic)

Binary -> M-valued (1WNR regular)

Code for process pi:

```
initially:
        shared array R[0,..M-1] of 1WNR registers := [1,0,…,0]

upon read()
        for j = 0 to M-1 do
            if R[j].read() = 1 then return j

upon write(v)  // if i=1
        R[v].write(1)
        for j=v-1 down to 0 do R[j].write(0)
        return ok
```

Binary -> M-valued (1WNR regular)

- Correctness:
  - ✓only the last or concurrently written value can be returned
- HW what if:
  - ✓for j=0 to v-1 do R[j].write(0)
  - ✓upon write(v) // if i=1
    for j=v-1 down to 0 do R[j].write(0)
    R[v].write(1)
    return ok

Transformations

From 1W1R binary safe to 1WNR multi-valued atomic

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Histories

A history is a sequence of invocation and responses

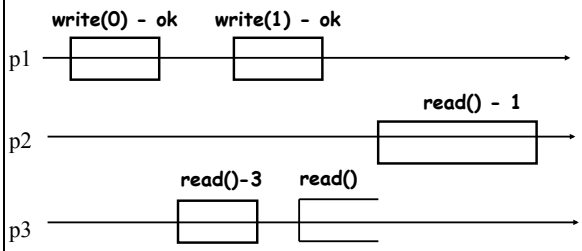
E.g., p1-write(0), p2-read(),p1-ok,p2-0,…

A history is sequential if every invocation is immediately followed by a corresponding response

E.g., p1-write(0), p1-ok, p2-read(),p2-0,…

(A sequential history has no concurrent operations)

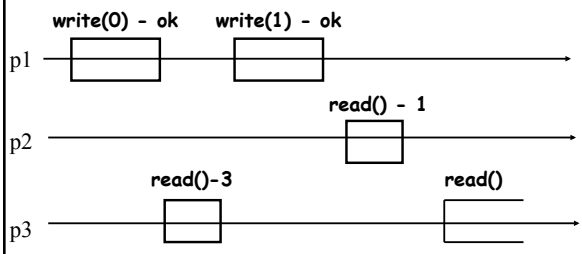
Histories



History:

p1-write(0); p1-ok; p3-read(); p1-write(1); p3-3; p3-read(); p1-ok; p2-read(); p2-1

Histories



History:

p1-write(0); p1-ok; p3-read(); p3-3; p1-write(1); p1-ok; p2-read(); p2-1; p3-read();

Legal histories

A sequential history is *legal* if it satisfies the sequential specification of the shared object

Read-write registers:

Every read returns the argument of the last write

(well-defined for sequential histories)

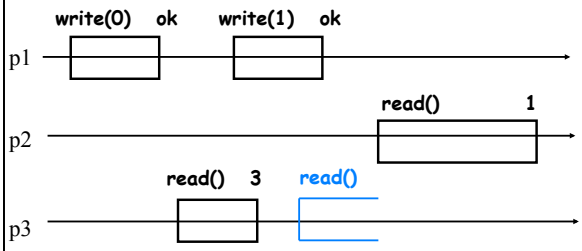
Complete operations and completions

Let H be a history

An operation op is *complete* in H if H contains both the invocation and the response of op

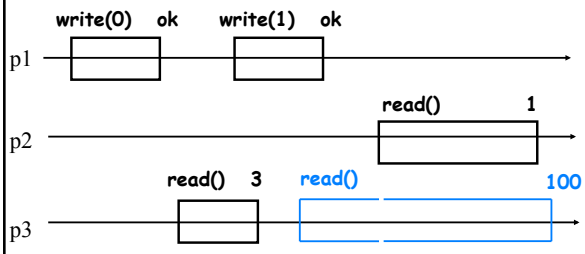
A *completion* of H is a history H' that includes all complete operations of H and a subset of incomplete operations of H followed with matching responses

Complete operations and completions



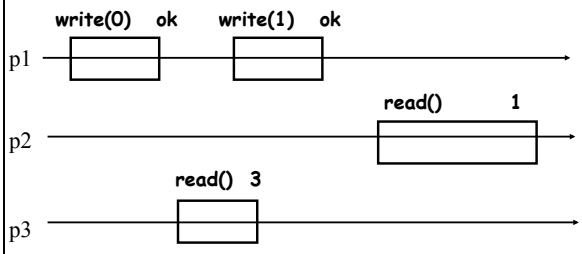
p1-write(0); p1-ok; p3-read(); p1-write(1); p3-3; p3-read(); p1 -ok; p2-read(); p2-1;

Complete operations and completions



p1-write(0); p1-ok; p3-read(); p1-write(1); p3-3; p3-read(); p1 -ok; p2-read(); p2-1; p3->100

Complete operations and completions



p1-write(0); p1-ok; p3-read(); p1-write(1); p3-3; p1-ok; p2-read(); p2-1

Equivalence

Histories H and H' are *equivalent* if for all pi

$H|p_i = H'|p_i$

E.g.:

H=p_1-write(0); p_1-ok; p3-read(); p_3-3

H'=p_1-write(0); p_3-read(); p_1-ok; p_3-3

Linearizability (atomicity)

A history H is *linearizable* if there exists a legal sequential history S such that:

- S is equivalent to some completion of H
- S preserves the precedence relation of H:

op1 precedes op2 in H => op1 precedes op2 in S

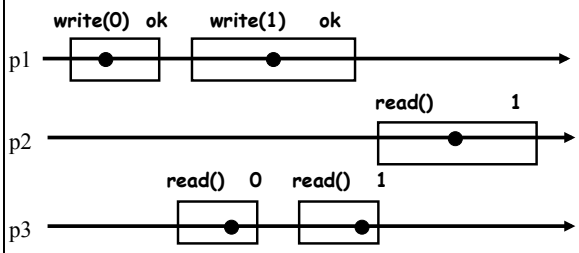
Atomic register

A register is *atomic* if every history it produces is linearizable

Informally, the complete operations (and some incomplete operations) are seen as taking effect instantaneously at some time between their invocations and responses

(The operations are *atomic*)

Atomic?



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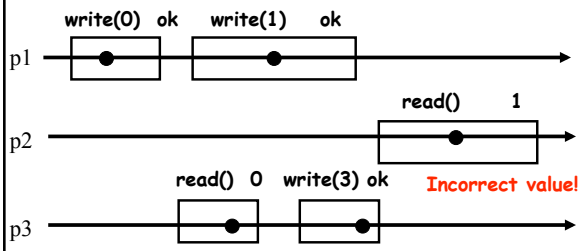

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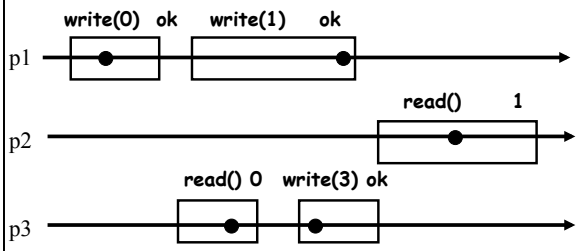

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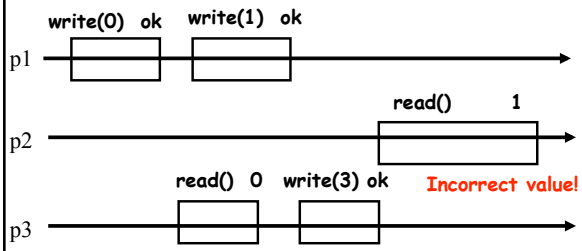

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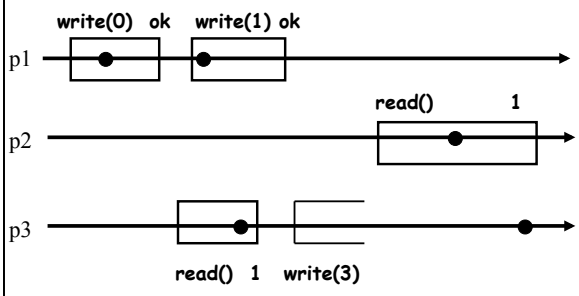

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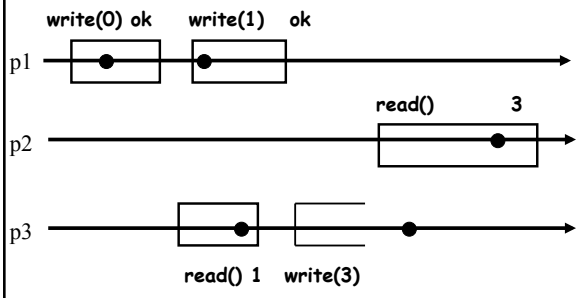

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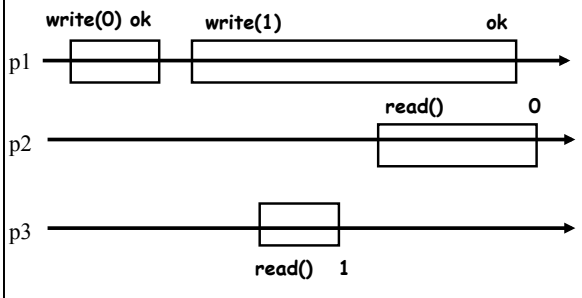
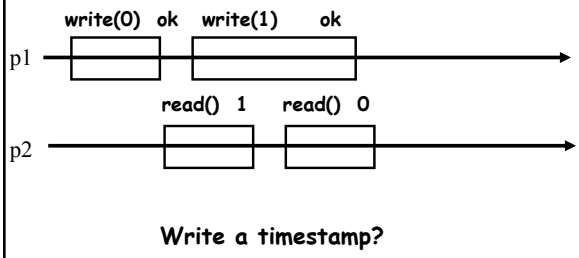


From 1W1R regular to 1W1R atomic



Write a timestamp?

1W1R regular -> 1W1R atomic

Code for process pi:

initially:
- shared 1W1R regular register R := 0
- local variables t := 0, x := 0

upon read()
- (t',x') := R.read()
- if t' > t then t:=t'; x:=x';
- return(x)

upon write(v) // if i=1
- t:=t+1
- R.write(t,v)

Transformations

From 1W1R binary safe to 1WNR multi-valued atomic

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Transformations-I

From safe to regular (binary 1W1R)

- Writer touches shared memory only to change
- A concurrent read is allowed to return any value (0 or 1)

Transformations-II

From one-reader to multiple-reader (regular binary or multi-valued)

- Every reader is assigned a dedicated register to read
- Writer writes in all
- Reader reads its own register

Transformations-III

From binary to M-valued (1WNR regular)

- Every *value* in {0,…,M-1} is assigned a dedicated 1WNR register
- Write(v) sets R[v] to 1 and sets R[v-1] … R[0] to 0
- Read returns the smallest v such that R[v]=1

Transformation IV

From regular to atomic (1W1R multi-valued)

- Write a timestamp with a value
- The reader returns the latest value and ignores the old one

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From regular to atomic (1W1R multi-valued)

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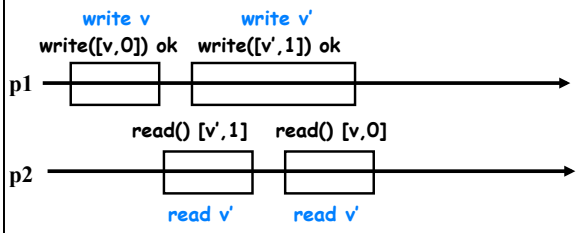
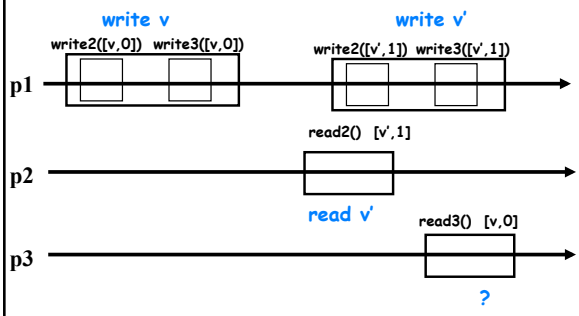


Multiple readers?

Does not work either!



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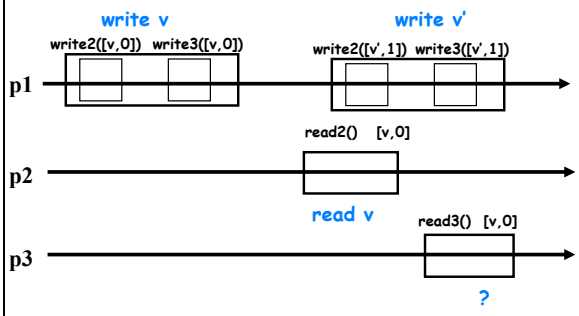

Transformation V

We use:
matrix RR[1..N][1..N] of 1W1R atomic registers := 0^{NxN}
// for all i,j, RR[i][j] is read by pi and written by pj

array WR[1..N] of 1W1R atomic registers := 0^{N}
// for all i WR[i] is written by p1 and read by pi

upon write(v) // code for p1
ts:=ts+1
for all j do WR[j].write([v,ts])
return ok

Transformation V

upon read() // code for pi
for all j=1,…,N do (t[j],x[j]) := RR[i][j].read()
(t[0],x[0]) := WR[i].read()
(t,x) := highest(t[..],x[..])
for all j do RR[j][i].write([t,x]);
return(x)

Transformation V

If read1 returns v and read1 precedes read2 then read2 cannot return a value that is older than v – sufficient for linearizability in the one-writer case

HW:
- What if readers don't write?
- Multiple writers?