FDS 12 Problem Set 5:
Consensus

Exercise 5.1: critical run

Consider any algorithm A that solves binary consensus in the wait-free manner using any collection of atomic objects. We assume that the objects are deterministic and, thus, any run of A is unambiguously determined by its initial state and the sequence of process identifiers specifying the order in which the processes take steps.

A finite run is called *v-valent* ($v \in \{0, 1\}$) if v is the only value decided in extensions of R. A run is *bivalent* if it has an extension in which 0 is decided and an extension in which 1 is decided.

Show that A has a *critical* run, i.e., a bivalent run R such that for all p_i, $R.i$ (extension of R with one step of p_i) is *univalent* (0-valent or 1-valent).

Hint: By contradiction. Start with any bivalent empty run (e.g., the one in which p_0 proposes 0 and p_1 proposes 1) and consider longer and longer bivalent extensions of it.

Exercise 5.2: three-process consensus with queues

Show that queues and registers are not strong enough to solve consensus among three or more processes.
Hint: Consider a critical run R such that the decision value in any extension of it is defined by the state of some queue object: e.g., $R.0$ is 0-valent, $R.1$ is 1-valent, and both p_0 and p_1 are *about* to access a queue object Q at the end of R.