FDS 12: Problem Set 3:
Atomic and Immediate Snapshot

A (one-shot) immediate snapshot (IS) object exports one operation $WriteRead_i(v)$ to every process p_i that takes a value as an argument and returns a vector S of N values, one value per process in response. In each run, IS ensures the following properties (here v_i denotes the value proposed by process p_i and S_i denotes the vector returned to p_i in that run):

- *Self-inclusion*: for all i, $v_i \in S_i$
- *Containment*: for all i, j: $S_i \subseteq S_j \vee S_j \subseteq S_i$
- *Immediacy*: for all i, j: if $v_i \in S_j$, then $S_i \subseteq S_j$

Exercise 3.1

Prove that any run of one-shot Atomic snapshot (every process p_i first performs $update_i(v_i)$ and then $S_i := snapshot()$) satisfies the self-inclusion and containment properties.

Exercise 3.2

Show that the vectors returned in each run of IS correspond to some block run of one-shot AS (see slide 27 of Class 5) and, vice versa, the vectors returned in each block run of one-shot AS correspond to some run of IS.

Similarly, prove that the *standard chromatic subdivision* (slide 30) of the initial 2-*simplex* indeed captures all possible IS runs. We need to show that every run of IS corresponds to a simplex in the subdivision and, vice versa, every simplex in the subdivision corresponds to an IS run. Note that we should consider all simplices in the subdivision: 0-dimensional (individual vertices), 1-dimensional (intervals), and 2-dimensional (triangles).