FDS 12: Problem Set 2:
Atomic-bit construction

Exercise 2.1: reconstructing of a counter-example

In the class, we considered the following (*almost* correct) algorithm (Iteration 3 in the slides):

operation $write(v)$: % Invoked only to change the value %
i change REG;
ii **if** $WR = RR$ **then** change WR; % Strive to establish $WR \neq RR$ %

operation $read()$:

1 **if** $WR = RR$ **then** $return\ (val)$;
2 $aux := REG$; % Conservative value %
$3'$ change RR; % Strive to establish $WR = RR$ %
4 $val := REG$;
5 **if** $WR = RR$ **then** $return\ (val)$;
7 $return\ (aux)$

Construct a run of this algoirithm in which new-old inversion is observed (following the hints in the slides).

Exercise 2.2: Proof of Tromp's construction

Prove that the algorithm below indeed implements an atomic bit.

operation $write(v)$: % Invoked only to change the value %
i change REG;
ii **if** $WR = RR$ **then** change WR; % Strive to establish $WR \neq RR$ %

operation $read()$:
1 **if** $WR = RR$ **then** $return\ (val)$;
2 $aux := REG$;
3 **if** $WR \neq RR$ **then** change RR;
4 $val := REG$;
5 **if** $WR = RR$ **then** $return\ (val)$;
6 $val := REG$;
7 $return\ (aux)$

Hint: prove that for the reading function π defined in the slides satisfies:

$A0$: $\forall\, r$: $\neg(r \rightarrow_H \pi(r))$. *(No read returns a value not yet written.)*

$A1$: $\forall\, r, w$ in H: $(w \rightarrow_H r) \Rightarrow \big(\pi(r) = w \ \vee\ w \rightarrow_H \pi(r)\big)$. *(No read obtains an overwritten value.)*

$A2$: $\forall\, r1, r2$: $(r1 \rightarrow_H r2) \Rightarrow \big(\pi(r1) = \pi(r2) \ \vee\ \pi(r1) \rightarrow_H \pi(r2)\big)$. *(No new/old inversion.)*