# FDS : Exercise 6
# Asymptotic Notations and Graph Terminologies

**NOTE: Exercise 6 will not be evaluated and is not considered for the grades**

(1) In this question, we recall the definitions of the Asymptotic notations.

Let $f$, $g$: $\mathbb{N} \to \mathbb{N}$.

We say $f$ is $O(g)$ if $\exists c \in \mathbb{N}$ and $n_0$ such that $\forall n \geq n_0$, $f(n) \leq c \cdot g(n)$.

We say $f$ is $o(g)$ if $\forall c \in \mathbb{N}$, there is $n_0$ such that $\forall n \geq n_0$, $f(n) \leq c \cdot g(n)$ i.e. $f$ grows strictly more slowly than *any* arbitarily small positive constant multiple of $g$.

We say $f$ is $\Omega(g)$ if $g$ is $O(f)$.

We say that $f$ is $\Theta(g)$ if $f$ is both $O(g)$ and $\Omega(g)$.

Now answer the following questions and explain your solutions in detail.

(1a) Let $f_1(n) = 23n^2$. Is $f_1(n) \in o(n^3)$?

(1b) Let $f_2(n) = 42n^3 + n^2 \log n$. Is $f_2(n) \in \Theta(n^3)$?

(1c) Is $3^{\log n} \in O(n)$?

(1d) Consider the geometric summation $\sum_{i=0}^{n} \frac{1}{2^i}$. Is it $O(1)$?

(1e) Consider the problem of sorting a sequence of $n$ distinct elements from a set $S$ such that given any two elements in $S$, we can compare them (for example, you can think of $S$ to be a finite set of natural numbers). This is the only operation available to gain information about the sequence. Show that any algorithm that uses sorts by comparisons on some sequence of length $n$ must use at least $c \cdot n \log n$ comparisons for some constant $c$.
Hint: $n! \geq (\frac{n}{2})^{\frac{n}{2}}$

(2) (2a) Recall the definition of *perfect matching* (Definition 5.15 in the Lecture Notes). Show that if a Graph $G(V, E)$ has a perfect matching, then $|V|$ is *even*.

(2b) Recall the definition of *independent sets* (Definition 5.1 in the Lecture Notes). Describe a *greedy algorithm* (sequential algorithm) for computing an *maximal independent set (MIS)* in a given graph. Argue biefly why it is correct.