

Homework 4

(Network Optimization by Randomization)

1. (20 pts.) Consider the following experiment: pick one number, uniformly, from the set $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Consider the sets $A = \{1, 2, 3, 4\}$ and $B = \{2, 3, 4\} \cup C$, for some set C . Determine C such that the events A and B are

- (a) Independent
- (b) Non-independent

2. (30 pts.) Consider the sample space $\Omega = \{abc, acb, cab, cba, bca, bac, aaa, bbb, ccc\}$ and assign the probability measures $\mathbb{P}(\{\omega\}) = \frac{1}{9}$, $\forall \omega \in \Omega$. Let the events

$$A_k = \{\text{the } k^{\text{th}} \text{ letter is } a\}, \quad k = 1, 2, 3.$$

- (a) Describe (in words) the underlying experiment corresponding to Ω .
 - (b) Show that the events $\{A_1, A_2, A_3\}$ are pairwise independent but not independent.
3. (20 pts.) Consider two independent events A and B . Comment on the independence of the following events
- (a) A and B^c
 - (b) A^c and B^c

4. (30 pts.) Consider the following experiment: you throw two fair dice. Determine whether the event

$$A = \{\text{the first die shows } 4\}$$

is independent, or not-independent, of each of the following two events

- (a) $S_5 = \{\text{the sum of the dice is } 6\}$
 - (b) $S_7 = \{\text{the sum of the dice is } 7\}$
5. (Bonus: 25 pts.) Continuation of Problem 4. Would your conclusions change if the dice were not fair?

Note: Homework due on May 26th.