Homework 3

(Network Optimization by Randomization)

1. (30 pts.) You perform the following experiment: you randomly throw 3 balls (a, b, and c) into two bins (x and y). Construct two distinctive probability spaces for the experiment (make sure that you state all your assumptions!).

2. (40 pts.) Consider a bin containing three balls (one "Red" and two "Blacks"), and assume that you make the following experiments:

   (a) You randomly extract two balls sequentially *without* replacement, i.e., you don't put back extracted balls.

   (b) You randomly extract two balls sequentially *with* replacement, i.e., once you extract a ball you write down its color and put it back before extracting the next.

   For each experiment construct a probability space (as in Problem 1, make sure that you state all your assumptions!).

3. (30 pts.) Let A and B two events on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$, and assume that $\mathbb{P}(A) = \frac{3}{4}$ and $\mathbb{P}(B) = \frac{1}{3}$.

   - Show that
   $$\frac{1}{12} \leq \mathbb{P}\left(A \cap B\right) \leq \frac{1}{3} \, ,$$
   and also give examples (of the probability space, together with A and B) showing that both the upper and lower bounds (i.e., $\frac{1}{12}$ and $\frac{1}{3}$) are possible.
   - Find upper and lower bounds for $\mathbb{P}\left(A \cup B\right)$.

4. (Bonus: 25 pts.) Assume that you have available a fair coin. Write a procedure to generate the outcome of a biased coin, i.e., $\mathbb{P}(\text{Heads}) = 1 - p$ and $\mathbb{P}(\text{Tail}) = p$. What is the expected running time of your algorithm?

   Note: Homework due on May 19^{th}.