Homework 2

(Network Optimization by Randomization)

Note: You can solve this homework in groups of at most two students (only one submission per group).

1. (30 pts.) Consider the problem of verifying the consistency of two versions x and y of a database, as described in Lecture 2. Assume that both x and y consist of alternating sequences of 1's and 0's, starting with the 1's, and where the number of elements in each maximal sequence (of 1's or 0's) is a power of 2 and is also greater or equal than 4. For instance: $x = 11110000111111110000$ is valid but $x = 11110011111111$ is invalid (the first sequence of 0's has less than 4 elements), $x = 00001111$ is invalid (all the sequences must start with 1's), and $x = 11110000001111$ is also invalid (the first sequence of 0's has 6 elements which is not a power of 2). Give a deterministic communication protocol for consistency verification by sending *less* than n bits, where $n = 2^k$ is the number of bits in both x and y, and $k \geq 2$. What is the worst case communication cost (maximal number of bits to be sent)? What is the best case communication cost (minimal number of bits to be sent)?

2. (20 pts.) Continuation of Problem 2. Assume that $n = 2^4$, the sequences of 1's and 0's have either 4 or 8 elements, each occurring with probability $\frac{1}{2}$ (i.e., x is built by repeatedly flipping a fair coin; for example, if *head*(H) stands for 4 and *tail*(T) stands for 8 then the event HHT generates $x = 1111000011111111$). What is the average communication cost?

3. (30 pts.) Design your own pseudo-random number generator (PRNG) to uniformly generate numbers in the interval $[1, 10^3]$, using your favorite programming language. Argue whether the generated numbers are "sufficiently random" or not. What is the maximal and minimal periods that you could get? Show the first 20 numbers (if at least as many) for the sequences with maximal/minimal periods.

4. (20 pts.) Continuation of Problem 3. Use your own PRNG to simulate 10^6 tossings of a fair coin. What is the proportion of "heads(H)" in the first 10^i tossings for $i = 1, 2, \ldots, 6$. Do you have sufficient scientific evidence that your own PRNG is a good one?

5. (Bonus: 25 pts.) Assume that you have available a fair coin. Write a procedure to generate a permutation of $(1, 2, \ldots, n)$ according to the discrete uniform distribution, i.e., the probability of any outcome is $\frac{1}{n!}$.

Note: Homework due on May 12^{th}.