1 Leader Election in an “Almost Anonymous” Ring

a) Is deterministic leader election possible in a synchronous ring in which all but one processors have the same identifier? Either give an algorithm or prove an impossibility result.

b) Is deterministic leader election possible in a synchronous ring in which all but two processors have the same identifier? Either give an algorithm or prove an impossibility result.

2 Distributed Computation of the AND

Consider an anonymous ring where each processor has a single bit as input. You can assume that nodes can distinguish between their neighbors, i.e., when a node $v$ receives a message, $v$ knows which neighbor has sent the message (note that nodes may not know a consistent clockwise or counterclockwise orientation of the ring!).

a) Prove that there is no uniform synchronous algorithm for computing the AND of all input bits.

b) Present an asynchronous (non-uniform) algorithm for computing the AND; the algorithm should send $O(n^2)$ messages in the worst case.

c) Present a synchronous (non-uniform) algorithm for computing the AND; the algorithm should send $O(n)$ messages in the worst case. What is the time complexity of your algorithm?