Self-similar traffic

- Measured Data Traffic (Ethernet LAN)
  - Time Unit = 100 Seconds
  - Time Unit = 10 Seconds
  - Time Unit = 1 Second
  - Time Unit = 0.1 Second
  - Time Unit = 0.01 Second

- Traditional Models for Data Traffic
  - Time Unit = 100 Seconds
  - Time Unit = 10 Seconds
  - Time Unit = 1 Second
  - Time Unit = 0.1 Second
  - Time Unit = 0.01 Second
Self-similarity
Aggregate traffic - exact self-similarity

Intuition: self-similar processes “look the same” at all (i.e., over a wide range of) time scales

Def.: A stationary process \( X = (X_k : k \geq 1) \) is called exactly self-similar (with self-similarity parameter \( H, 0 < H < 1 \)), if for all \( m \geq 1 \),

\[
X = m^{1-H} X^{(m)}
\]

[LTWW94] LAN traffic is consistent with exact self-similarity
Aggregate traffic - exact self-similarity

Intuition: self-similar processes “look the same” at all (i.e., over a wide range of) time scales

Def.: A stationary process $X = (X_k : k \geq 1)$ is called exactly self-similar (self-similarity parameter $H$, $0 < H < 1$), if for all $m \geq 1$,

$$X = m^{1-H} X^{(m)}$$

$$\text{var}(X^{(m)}) \sim cm^{-2H-2} \quad \text{as} \quad m \to \infty$$
Variance time plot
Network topology 1989
Network topology 1992
Self-similarity

- Just a mathematical concept?
- What does it mean?
Self-similarity via heavy tails

Math:

Superposition of independent ON/OFF sources is self-similar, if durations of periods are heavy-tailed with infinite variance

Superposition of independent ON/OFF sources is short-range dependent, if durations of periods are light-tailed
Superposition of sources
Covariance

Given two random variables \( x, y \) with means \( \mu_x \) and \( \mu_y \), their covariance is:

\[
\text{Cov}(x, y) = \sigma_{xy}^2 = \mathbb{E}[(x - \mu_x)(y - \mu_y)] = \mathbb{E}[xy] - \mathbb{E}(x)\mathbb{E}(y)
\]

Their correlation coefficient is the normalized covariance

\[
\text{Cor}(x, y) = \rho_{xy} = \frac{\sigma_{xy}^2}{\sigma_x \sigma_y}
\]
Short-Range Dependence

A stationary process $X = (X_k : k \geq 1)$ with mean $\mu$, variance $\sigma^2$ and autocorrelation function $X r(k), k \geq 1$, is said to exhibit short-range dependence (SRD) if there exists $0 < \rho < 1$ and $\tau > 0$ with

$$r(k)\tau \rho^{-k} \to 0 \text{ as } k \to \infty$$

Important feature: Autocorrelations decay (at least) exponentially fast for large lags $k$
Poisson process: a SRD processes
Short-range dependence

- The aggregated process $X^{(m)} = (X^{(m)}(k); k \geq 1)$ tends to second-order white noise, as $k \to \infty$

$$r^{(m)}(k) \to 0 \quad \text{as} \quad k \to \infty$$

for all $k \geq 1$, where $r^{(m)}$ denotes the autocorrelation function of $X^{(m)}$

- The variance-time function, i.e., the variance of the sample mean, as a function of $m$, satisfies:

$$\text{var}(X^{(m)}) \sim c m^{-1} \quad \text{as} \quad m \to \infty$$
Short-range dependence

Key features

- Short range dependence = finite correlation length
- Fluctuations over narrow range of time scales
- Plotting $\text{var}(X^{(m)})$ vs. $m$ on log-log scale shows linear relationship for large $m$, with slope $-1$
Light-tailed distributions

- X random variable with distribution function F.
- F is said to be light-tailed if there exists $c > 0$
  \[
  (1 - F(X))e^{cx} \to 0 \quad \text{as} \quad x \to \infty
  \]
- Important feature: tails decay exponentially fast for large $x$; i.e.,
  \[
  P[X > x] = 1 - F(X) \sim e^{-x} \quad \text{as} \quad x \to \infty
  \]
Light-tailed distributions

- Examples: Exponential, Normal, Poisson, Binomial

- Key features:
  - \( F \) has limited variability
  - \( F \) is tightly concentrated around its mean
  - \( F \) has finite moments
  - \( P[X > x] \) vs. \( x \) on log-linear scale is linear for large \( x \)
Summary of light-tails and SRD

- Distributional assumptions
  - Light-tails imply limited variability in space
- Assumptions about temporal dynamics
  - SRD implies limited variability over time
- Common characteristics of traditional traffic processes
  - Limited burstiness (in time and space)
Long-range dependence

- A stationary process $X = (X_k : k \geq 1)$ with mean $\mu$, variance $\sigma^2$ and autocorrelation function $X r(k)$, $k \geq 1$, is said to exhibit long-range dependence (LRD) if for some $1/2 < H < 1$ and

  $$r(k) \sim c k^{2H-2} \quad \text{as} \quad k \to \infty$$

$H$ is called the Hurst parameter.

- Important features of LRD
  - Infinite correlation length
  - Fluctuations over all time scales
  - No characteristic time scale
Long-range dependence

- The aggregated process $X^{(m)} = (X^{(m)}(k); k \geq 1)$ tends to non-degenerate limiting process, for $m, k$ sufficiently large

$$r^{(m)}(k) \rightarrow r(k) \quad \text{as} \quad k \rightarrow \infty$$

- The variance-time function satisfies:

$$\text{var}(X^{(m)}) \sim cm^{2H-2} \quad \text{as} \quad m \rightarrow \infty$$
Heavy-tailed distributions

- X random variable with distribution function F
- F is said to be heavy-tailed if there exists $c > 0$

$$1 - F(X) = P[X > x] \sim cx^{-\alpha} \quad \text{as} \quad x \to \infty$$

- Important features:
  1. $1 < \alpha < 2$, $X$ has finite mean but infinite variance
  2. Heavy-tailed implies high variability
  3. Tail decays like a power, hence power-law dist.
  4. Plotting $P[X > x]$ vs. $x$ on log-log scale is linear for large $x$ with slope $\alpha$
Detour
Characteristics of modem calls
(\sim1999)
Interarrival times of modem calls
Durations of modem calls
What about pkts from modem calls
Detour
Characteristics of Web
(∼2000)
General characteristics of WWW transfers
General characteristics of WWW transfers
General characteristics of WWW transfers

![Graph showing number of HTTP requests over time.](image)
# of TCP connections per session
Flow durations
Why is LAN traffic self-similar

Possible explanations:
- Network?
- User behavior?

User behavior:
- Examine characteristics of individual src-dst pairs
- Clustering of packets between src-dst pairs
- Define clusters as ON/OFF periods
- Distribution of ON/OFF periods
SRC/DST traffic matrix
Texture plot
Texture plot
Grouping IP packets into flows

- Group packets with the “same” address
  - Application-level: single transfer web server to client
  - Host-level: multiple transfers from server to client
  - Subnet-level: multiple transfers to a group of clients

- Group packets that are “close” in time
  - 60-second spacing between consecutive packets
ON/OFF periods
ON/OFF periods are heavy-tailed
Self-Similarity via heavy tails

Math:
Superposition of independent ON/OFF sources is self-similar, if durations of periods are heavy-tailed with infinite variance

Statistical analysis of LAN traffic traces:
- Users are ON/OFF
- ON periods are heavy-tailed (file sizes)
- OFF periods are heavy-tailed (think times)
- Distributions of ON/OFF-periods show heavy tails with infinite variance
Wide area network traffic

How are WANs different from LANs
- Network effects matter: roundtrip delays, queuing, flow control
- Many more source destination pairs (not continuously active)

WAN traffic is not exactly self-similar [PF95, FGWK98]
- Generalize notion of self-similarity
- Examine nature of traffic at application/connection layer
- Beyond self-similarity (where are the network effects)
Asymptotic self-similarity

Def.: A stationary process \( X = (X_k : k \geq 1) \) is called asymptotically self-similar (with self-similarity parameter \( H, 0 < H < 1 \)), if for all large enough \( m \),

\[
X \approx m^{1-H} X^{(m)}
\]

Observations:

- Asymptotic self-similarity is equivalent to long-range dependence of infinite correlation length
- Asymptotic self-similarity does not specify the small-time scale behavior of a process
Structural model of WAN traffic

Cox’s construction

- M/G/oo model or birth-immigration process
  - Poisson session arrivals
  - Session durations or session sizes are heavy tailed with infinite variance (i.e., $1 \leq \alpha < 2$)
  - Traffic within session is generated at constant rate

- The resulting process is (asymptotically second-order) self-similar with self-similarity parameter

$$H = \frac{(3 - \alpha)}{2}$$
Structural model of WAN

- Telnet and FTP sessions
  - Extract session-level information from WAN traces
  - Test if arrivals are consistent with Poisson
  - Test if arrivals are consistent with independence
Dataset WAN traffic LBL/WRL
Test for Poisson arrivals
Test for heavy tail
Implications (shaded 2%, black 0.5%)
Self-similar?
Self-similar?
Mathematical results

LAN:
- Superposition of independent ON/OFF sources
- ON/OFF periods are heavy-tailed with infinite variance
  Packets per unit time is exactly self-similar

WAN:
- Sessions arriving in a Poisson manner
- sizes (# packets) are heavy-tailed with infinite variance
  Packets per unit time is asymptotically self-similar
Statistical analysis of WEB

Before Web (1994):
- **Self-similarity at packets per time unit**
  - Poisson arrivals at application layer (FTP, Telnet)
  - Heavy-tailed session durations/sizes

Since Web (1995)?
- Arrivals of User session
- # of Web requests per session
- Dist. of # of bytes, pkts, duration per request?
Web client trace analysis 1995

- Modified Web browser (Mosaic)
- Population: students at BU
- Duration: 21 Nov 94 to 8 May 95

<table>
<thead>
<tr>
<th>Category</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Sessions</td>
<td>4,700</td>
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<tr>
<td>Users</td>
<td>591</td>
</tr>
<tr>
<td>URLs Requested</td>
<td>575,775</td>
</tr>
<tr>
<td>Files Transferred</td>
<td>130,140</td>
</tr>
<tr>
<td>Unique Files Requested</td>
<td>46,830</td>
</tr>
<tr>
<td>Bytes Requested</td>
<td>2,713 MB</td>
</tr>
<tr>
<td>Bytes Transferred</td>
<td>1,849 MB</td>
</tr>
<tr>
<td>Unique Bytes Requested</td>
<td>1,088 MB</td>
</tr>
</tbody>
</table>
What about WEB traffic
Durations of WEB transfers?
File size of WEB transfers?
Unique files vs. files transferred?

![Graph showing the log-log plot of unique files, file transfers, and file requests against log-log file size in bytes.](image-url)
What about the available files?
What about off times?
What about the WEB
Interarrival times of URL requests
Statistical analysis of WAN traffic Traces

Before Web (1994):
- **Self-similarity at packets per time unit**
  - Poisson arrivals at application layer (FTP, Telnet)
  - Heavy-tailed session durations/sizes

Since Web (1995):
- **Self-similarity at # of TCP connections per time unit**
  - Poisson arrivals of User session (modem session)
  - Heavy-tailed # of TCP connections per session