FDS 12: Problem Set 4:
Immediate Snapshot and Iterated Immediate Snapshot

A (one-shot) immediate snapshot (IS) object exports one operation WriteRead\(_i\)(\(v\)) to every process \(p_i\) that takes a value as an argument and returns a vector \(S\) of \(N\) values, one value per process in response. In each run, IS ensures the following properties (here \(v_i\) denotes the value proposed by process \(p_i\) and \(S_i\) denotes the vector returned to \(p_i\) in that run):

- **Self-inclusion**: for all \(i\), \(v_i \in S_i\)
- **Containment**: for all \(i, j\): \(S_i \subseteq S_j \vee S_j \subseteq S_i\)
- **Immediacy**: for all \(i, j\): if \(v_i \in S_j\), then \(S_i \subseteq S_j\)

**Exercise 4.1**
Prove that the algorithm in slide 34 implements an IS (e.g., using the outline proposed in the slides).

**Exercise 4.2**
Would the algorithm be still correct if instead of \(A_r.\text{update}_i(\(v_i\))\) and \(S := A_r.\text{snapshot}()\) we use \(U_r[i].\text{write}(\(v_i\))\) and \(S := \text{scan}(U_r[1], \ldots, U_r[N])\) respectively, where \(U_r\) is a shared vector of \(N\) one-writer-\(N\)-reader atomic registers? Justify your judgement.

**Exercise 4.3**
Consider the simulation of (multiple-shot) AS in the iterated IS (IIS) model (slide 26 in Class 6). Complete the proofs of the following claims (e.g., using the outline proposed in the slides):

**Lemma 2** Let \(c_r\) and \(c'_r\) be the clock vectors evaluated by processes \(p_i\) and \(p_j\), resp., in round \(r\). Then \(|c_r| \leq |c'_r|\) implies \(c_r \leq c'_r\).

**Corollary 1** All processes that complete a snapshot operation in round \(r\), get the same clock vector \(c\), \(|c| = r\)

**Corollary 2** If a process completes a snapshot operation in round \(r\) with clock vector \(c\), then for each clock vector \(c'\) evaluated in round \(r' \geq r\), we have \(c \leq c'\).